MAC 2313, Section 06 with Dr. Hurdal
Fall 2009 – Final Exam

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. There will be 10 marks allocated for clear and well written mathematics solutions. This test will be graded out of 100.

1. (8 marks) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.

2. (15 marks) Verify Stokes' Theorem (meaning compute the line integral directly and using Stokes' Theorem) for the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented in the direction of the positive z-axis.

3. (5 marks) A particle starts at $(1, 2, -1)$ with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. Find its position function.

4. (10 marks) Show that $\mathbf{F}(x, y, z) = \langle e^y, xe^y + e^z, ye^z + 2\rangle$ is conservative and use this fact to evaluate the work done in moving a particle along the line segment from $(0, 2, 0)$ to $(4, 0, 3)$.

5. (7 marks) Evaluate $\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}(x^2z+y^2z+z^3)\,dz\,dx\,dy$ by changing to spherical coordinates.

6. (10 marks) Directly evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle \sin x, \cos y, xz \rangle$ and C is given by $\mathbf{r}(t) = \langle t^3, -t^2, t \rangle$ for $0 \le t \le 1$.

7. (15 marks) Consider the equation of the surface $z = \sqrt{3x^2 + 2y^2 - 3}$.
a) Sketch this surface.
b) Find the equation of the tangent plane to this surface at $(2, 0)$.
c) Find the directional derivative at $(2, 0)$ in the direction of $\mathbf{v} = \langle 3, 4 \rangle$.
d) What is the maximum rate of change at $(2, 0)$?

8. (10 marks) Use the Divergence Theorem to calculate the flux of $\mathbf{F}$ across S where $\mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

9. (10 marks) Evaluate $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$ by reversing the order of integration.

Bonus (10 marks): Green's Theorem states that if C is a positively oriented, piecewise smooth, simple closed curve in the xy-plane that bounds the region D and if $\mathbf{F}(x, y) = \langle P, Q, R = 0\rangle$ has continuous partial derivatives on an open region that contains D, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int\int_D (\text{curl } \mathbf{F}) \cdot \mathbf{k} \, dA.$$

Now, assume that C is a positively oriented, piecewise smooth, simple closed curve in the yz-plane that bounds the region D and $\mathbf{F}(y, z) = \langle P = 0, Q, R\rangle$ has continuous partial derivatives on an open region that contains D.

a) Obtain a formula similar to Green's Theorem that is for evaluating the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is in the yz-plane.

b) Use your result from a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the triangular curve consisting of the line segments from $(0, 0, 0)$ to $(0, 1, 0)$, from $(0, 1, 0)$ to $(0, 0, 1)$, and from $(0, 0, 1)$ to $(0, 0, 0)$ and $\mathbf{F}(y, z) = \langle 0, y^4, yz\rangle$.