MAC 2313, Section 03 with Dr. Hurdal
Fall 2003 Final Exam

Name: ______________________ SSN: ______________________

As stated in class, you are allowed to bring to the exam one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. This exam will be graded out of 100.

1. (12 marks) Evaluate $\int\int\int_B (x^2 + y^2)dV$, where B is enclosed by the planes $z = 0$ and $z = x + y + 3$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

2. (11 marks) Using 2 methods (direct computation and one of the fundamental integral theorems), compute the circulation of $\vec{F}$ around the circle of radius a that is sitting in the plane $z = 1$, with origin along the z-axis and oriented counter clockwise when viewed from above, where $\vec{F} = xz\vec{\imath} + (y - x)\vec{\jmath} + x\vec{k}$.

3. (10 marks) The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 + 3z^2$, where T is measured in degrees Celsius and x, y, z in meters.
(a) Sketch the level surface for the temperature when $T = 4°$.
(b) In which direction does the temperature increase fastest at the point $(0.6, 0.8, 1)$?
(c) What is the maximum rate of increase?
(d) Find an equation for the tangent plane to the level surface $T = 4$ at the point $(0.6, 0.8, 1)$.

4. (12 marks) Determine which of the following vector fields is a gradient field and state your reasoning. Then find a potential for that vector field.

(a) $\vec{F} = (2xyz^3)\vec{\imath} + (x^2z^3 + 2x)\vec{\jmath} + (3x^2yz)\vec{k}$

(b) $\vec{G} = (y^3z^2)\vec{\imath} + (3xy^2z^2 + 2yz^3)\vec{\jmath} + (2xy^3z + 3y^2z^2)\vec{k}$

5. (10 marks) A rectangular building is being designed to minimize heat loss. The east and west wall lose heat at a rate of 10 units/m^2 per day, the north and south walls at a rate of 8 units/m^2 per day, the floor at a rate of 1 unit/m^2 per day and the roof at a rate of 5 units/m^2 per day. The volume must be exactly 4000 m^3. Find the dimensions that minimize heat loss.

6. (10 marks) Two particles are traveling through space. At time t the first particle is at the point $(-1+t, 4-t, -1+2t)$ and the second particle is at $(-7+2t, -6+2t, -1+t)$. Do the two particles collide and/or intersect? If so, when and where?

7. (5 marks) Compute the flux of the vector field through the sphere $(x-2)^2+(y-3)^2+z^2 = 25$ oriented inward, where $\vec{F} = (e^y + x)\vec{\imath} + (y + z\cos(x^2))\vec{\jmath} + (z + x)\vec{k}$.

8. (10 marks) A particle is traveling along the curve $y = x^2 + 2x$ between $(0, 0)$ and $(2, 8)$. Compute the work done as it travels through the force field $\vec{F} = x^2\vec{\imath} + y\vec{\jmath}$.

9. (10 marks) Find the mass of the triangular metal plate with vertices at $(1, 0)$, $(0, 1)$ and $(1, 1)$ if the density is given by $\delta(x, y) = 1 + 3x + y$.

10. (10 marks) Identify and sketch the surface $\vec{r}(s,t) = 2\cos(s)\vec{\imath} + t\vec{\jmath} + 2\sin(s)\vec{k}$ for $0 \leq s \leq \dfrac{\pi}{2}$ and $-1 \leq t \leq 1$. Compute the flux of the vector field $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ through this surface if it has outward orientation.

Bonus (10 marks) To find the maximum and minimum values of a function $f(x, y, z)$ subject to the constraint $g(x, y, z) = c$, we used the method of Lagrange multipliers, where you find all values of x, y, z and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z),$$

i where λ is called a Lagrange multiplier. This method can be easily extended to 2 or more constraints. If you have 2 constraints, $g(x, y, z) = c$ and $h(x, y, z) = k$, you can find the maximum and minimum values of $f(x, y, z)$ by finding all values of x, y, z, λ and μ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) + \mu \nabla h(x, y, z),$$

where λ and μ are both Lagrange multipliers.
Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ subject to the constraints $g(x, y, z) = x - y + z = 1$ and $h(x, y, z) = x^2 + y^2 = 1$.

BLANK PAGE FOR EXTRA CALCULATIONS