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# Modelling War*

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A simple, general mathematical model for modern war is presented. The form of the model is Lanchester but its derivation owes approximately equal amounts to classical operational research and to later ideas associated with theoretical ecology – especially the concept of 'carrying capacity'. Solutions to the equations corresponding to stalemated, steady-state outcomes in theatre are concentrated on, with empirical justification derived from the unduly neglected databased work of Voevodsky. Prolongation and stalemate are seen as the default state of modern war. War termination is discussed as a consequence of 'mutually (but not equally) hurting stalemate'. Examples are given of how, in certain circumstances, stalemate may be pre-empted in theatre by striking at non-battlefield targets and light is cast on the late 20th-century strategic trend towards conducting war preponderantly from the air.

## Introduction

The principal objective of this article is to present a simple, general, mathematical model of war. It defines war as a contest of long or short duration involving armed clashes in one or more theatres between organized units which, in turn, rest upon the support (logistical, manpower) of a home base or bases, and which may be in theatre or outside, with the same home bases and their lines of linkage to forces in theatre also liable to armed attack.

The model is used to explain the long duration of a number of major wars in the past century and a half, and consequently is applicable to the question of war termination.[1] If the tendency of war is towards prolongation and 'mutually hurting stalemate', explanations of war settlement that take no account of war weariness are unlikely to be broadly applicable.[2] And, if stalemate is the natural condition of war, the question arises as to why it should be resorted to, if success (at best) lies a long way in the future.

The model is also used to explain the shifts in military strategy in the present century towards waging war off the battlefield as well as on it, as well as to shed light on the culmination of these shifts within the past decade (Gulf War 1990–91, Kosovo War 1999) for off-battlefield use of force to become the principal or even sole element in the conduct of the war.

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[1] For a survey of this question, which as a topic is probably not as neglected as its author claims, see Massoud (1996).

[2] For explanations that do take into account considerations of 'mutually hurting stalemate', see Mason & Flett (1996). For the origin of the idea, such as it is, that 'a ripe moment for the settlement [of a war] occurs when a mutually hurting stalemate develops', see Zartman (1985: 9).

## Model Building

As a starting point, it is self-evident that the net rate of increase of combat forces in the theatre of war must be the rate at which the home base introduces frontline personnel into the war, i.e. trains and equips and transports them, less the rate at which these forces are put out of action by the forces of the other side. Or rather, this would be true were we to ignore losses due to illness, desertions, etc., and this, for reasons of simplicity, we propose to do.

The next step is to determine the rate at which a home base (which we will call 'Red') can introduce forces into the theatre of war. At this point we make the important but reasonable assumption that there is an upper limit to the size of the armed forces Red's base or society can support in the field, which is determined by the size of Red's economy, Red's population (and its age distribution) and how far Red may be able to secure allies. For the moment we take this ceiling as fixed. Of course, this is also a simplification. Economies can grow, even in wartime. And war itself can induce direct changes. Not only may new allies be won over (or perhaps old ones defect), but economically valuable territory may be gained or lost by force of arms. Added to this, there is also deliberate enemy action to consider when it is specifically aimed at the infrastructure of Red's society.

It follows by definition that the rate at which Red can field new forces tends to zero as this upper limit is reached. The simplest mathematical relationship reflecting this situation puts the rate at which Red can introduce forces as proportional to the difference between the ceiling figure, $R_m$, and the size of the forces already present.

This particular approach to modelling the rate at which Red can introduce forces into the theatre of war makes indirect use of a concept borrowed from theoretical ecology – that of 'carrying capacity' which is the upper limit to a population of living things that may be supported by its environment.[3]

Modelling the rate at which Red's forces are put out of action by the other side (which we will call Blue) is more straightforward. Mainly on grounds of its simplicity, we adopt the classical Lanchester assumption, which sees this rate as directly proportional to the size of Blue's combat forces in theatre (Morse & Kimball, 1962: 72–73).

So we now have an equation for the net rate of increase of Red's forces in the theatre of war:

$$\text{Rate of increase (RED)} = l(R_m - r) - kb$$

$R_m$ is the ceiling figure for the size of Red's combat forces in theatre; $r$ is the size of Red's combat forces in theatre at any time ($t$): $b$ is the corresponding size of Blue's; $l$ is the coefficient of performance of Red's military supply arrangements (the training, equipping, transporting and support of frontline forces). It is the fraction of the eventual ceiling force that Red can put into the field, in unit time, at the start of the war, when the war is begun with far fewer troops at the front than the ceiling figure; and $k$ is the number of units of Red's forces a single unit of Blue's can account for (put out of action) in unit time. The units of force measurement are simply the frontline force manpower in theatre.

By symmetry, the analogous equation for Blue is:

$$\text{Rate of increase (BLUE)} = p(B_m - b) - qr$$

where $p$ is the parallel of $l$ and $q$ the parallel of $k$.

These two equations can be written in differential form as:

[3] For suggestive introductions to the concept of 'carrying capacity', see Brown & Rothery (1993: 27–31); May (1981: 80–81); and Smith (1968: 27).

$$dr/dt = lR_m - lr - kb \quad (1)$$

$$db/dt = pB_m - pb - qr \quad (2)$$

The justification for translating the left-hand side (rate terms) of the equations into a differential form is simply empirical. As will be seen, solutions of these *differential* equations, in terms of the size of in-theatre forces (*r*, *b*) over time, closely correspond to the recorded sizes of *r* and *b* over the time span of a number of major wars (*t*, the unit of time, could be measured in any convenient unit – e.g. weeks – and is taken to be zero at the start of the war).

### *Equations*

In this differential formulation, the form of the complete equations is already very familiar to students of quantitative approaches to war and peace issues. Apart from a sign, the equations are identical in form to the basic Richardson arms race equations. They are *formally* completely identical to what operational researchers refer to as generalized Lanchester equations, although the *interpretation* given here of the first two terms on the right hand side is quite new. And very similar equations appear in theoretical ecology as a description of the interaction of two species competing for the same environmental resources.

Correspondingly, what the solutions to these equations should look like is well explored. They fall into two classes, either steady-state solutions where $dr/dt$ and $db/dt$ are both zero and *r* and *b* possess some equilibrium pair of values, or non-steady-state solutions where either *r* or *b* go to zero even should their initial sizes correspond to the foregoing equilibrium values. Chaotic solutions, where *r* and *b* may take on pseudo-random values, do not arise, because a pair of simultaneous linear first-order differential equations never exhibits such behaviour. Interestingly, when we are faced with three such equations (with three variables, e.g. Red, Blue and Orange) chaotic solutions can arise (May, 1976).

The difference between the two aforementioned classes of non-chaotic solutions is essentially determined by the values of two sets of terms. When $kq < lp$, with $k/l$ and $q/p$ both small (i.e. < 1), normally a steady-state ensues with an equilibrium non-zero value for the sizes of the forces of both Red and Blue (see Appendix).

It is precisely these steady-state solutions that the present article regards as significant, for they are held to reflect accurately the nature of the major wars of the 20th century. We justify this statement with a three-layered argument.

The first layer is the apparent tendency for the major wars of this century to be prolonged beyond the duration time anticipated by a least one of the major participants. Obstinate prolongation, stalemate and 'resistance' to attempts to effect earlier termination are signs of a steady-state situation.

The second layer in the argument is semi-quantitative and comes from the pioneering (but unfortunately quite neglected) semi-analytical collection of war data assembled by Voevodsky (1971).

Voevodsky shows that the buildup of armies in the field (measured by the number of military personnel in frontline service in the appropriate theatre) for a number of major wars (the US Civil War, World Wars I and II, Korea and Vietnam), when plotted against time from the start of the conflict, follows precisely the sort of pattern one should expect from solutions to the above differential equations that envisaged steady-state outcomes (for the values of *r* and *b*). In other words he finds that the levels of forces deployed ($S(t)$ – *t* being elapsed time since the start of the war start) by the participants in these wars typically increase over time according to the equation:

$$S(t) = S_{ss}(1 - e^{-t/\tau}) \quad (3)$$

where $S_{ss}$ is the ultimate magnitude (e.g. steady-state number of men) of forces engaged in the theatre(s) of war as the duration of the war extends indefinitely (in actual wars, Voevodsky finds using curve-fitting that forces deployed, even if they do not reach $S_{ss}$ because the war finishes first, nonetheless tend towards $S_{ss}$); $\tau$ (tau) is a constant term with the dimensions of time and in an equation of this type usually called the 'time constant'. When $t = \tau$, it follows from Equation 3 that $S = S_{ss}$ [(e – 1)/e], or $0.63S_{ss}$.

Voevodsky explores this relationship in a number of interesting ways, but chooses to go only so far into the question of how it arises.[4]

But we can see how it arises quite easily by recourse again to Equations (1) and (2). If we first simplify the two equations for illustrative purposes by assuming that $k = q$ and $l = p$, and add the equations together we obtain:

$$d(r + b)/dt = l(R_m + B_m) - (k + l)(r + b) \quad (4)$$

In the steady-state, the combined size of the forces in the field approaches asymptotically to:

$$l(R_m + B_m)/(k + l) \quad (5)$$

If we call this total $S_{ss}$, and $(r + b)$, $S(t)$, and we introduce the term $S(0)$ for the combined size of the forces at time $t = 0$ (the war's beginning), then the solution of Equation 4, $S(t)$, is given by:

$$e^{-(k+l)t} = [S_{ss} - S(t)]/[S_{ss} - S(0)]$$

And when the initial ($t = 0$) size of the forces is very small in comparison with $S_{ss}$ (of course, not always necessarily the case), the above reduces to:

$$S(t) = S_{ss}(1 - e^{-(k+l)t}) \quad (6)$$

This is obviously the same time-dependence for the size of warring forces that Voevodsky identifies (equation 3). The value of $(k + l)$ – we can call it the inverse of the time-constant, $\tau$, for the wars – varies, according to Voevodsky, somewhat from war to war and combatant to combatant, but in no obviously regular manner. Its order of magnitude is nonetheless approximately 1 (year$^{-1}$).

The third layer of argument on behalf of the model of war presented here is quantitative, but merely suggestive. If it could be shown that the key inequality – $kq < lp$ ($k/l$ and $q/p$ both small) – held for all five major wars, the case for the model would be as completely made as such cases can ever be. Unfortunately, a comprehensive demonstration for all five wars is nigh impossible, essentially because of difficulties over data.

### *World War I*

However such a calculation is possible for World War I. This is because there are reasonably reliable figures available for the forces committed and uniformed casualties suffered by both sides or at least by important elements on both sides of the war. But it is also because it is the last statistically well-documented war in which military aircraft played only an insignificant part (by the standards of later wars) beyond the theatre of

[4] 'This ... analysis is based on a purely descriptive examination of the data ... of modern nations at war ... during the last 100 years. No attempt is made to explain the processes which result in the behavioral orderliness. The data on all five wars [American Civil, World War I, World War II, Korea, Vietnam] are seen to fit very closely the same, simple, mathematical laws' (Voevodsky, 1971: 145). On the other hand (p. 161) he brings to curve-fitting the prior assumption that '... the behavior of warring nations can be characterized by linear second-order feedback-control systems equations of the type first conceived by Wiener and supported by the work of Richardson, Lanchester and others ...'. Voevodsky on war resembles Kepler on the motion of the planets. Accurate data are presented and interim generalizations based on the data are also put forward, but no attempt is made explicitly to unify these generalizations under the single theme, à la Newton, of a model of war.

war itself (the relevance of this point will become clear).

This is not to say that even here a precise procedure is possible. First it is necessary to assume again that $k = q$ and $l = p$ (essentially positing a qualitative similarity between the warring sides in this war). To calculate $k$, we begin by integrating both sides of Equation (4) from time $t = 0$ to $t = \tau$ which gives, after some manipulation:

$$S(\tau) - S(0) = S_{ss} - (k + l)\,C(\tau)/k \qquad (7)$$

In Equation 7, $S(\tau)$ is the size of British, French and German in theatre forces on the Western Front at time $\tau$ after the war's start. $S(\tau)$ is by definition (see above) approximately $0.63S_{ss}$. Voevodsky's figure for $\tau$, the time constant for this war, $(k + l)^{-1}$, is approximately 1.3 years (he also estimates $S_{ss}$ for the British army in France at 2,000,000). $C(\tau)$ is the cumulative permanent losses for both sides up to time $\tau$. These comprise deaths, plus combatants wounded too seriously to take further part in the war, plus those made prisoner of war plus those missing.

Provided the data from the Western Front are known, this equation can be used to calculate $k$ (and hence $l$). Fortunately the additive nature of this equation makes this task a great deal easier since the relationships it cites as holding for the whole (of the Anglo–French alliance and their German opponents in France) hold for each of the constituent parts. And the easiest constituent part to deal with is the Anglo–French side for the main reason that, unlike the Germans, they fought essentially only on one front throughout.

We have from Voevodsky a figure for the British $S_{ss}$ at 2 million men. The French $S_{ss}$ has to be estimated from the fact that in the mature phase of the war French frontline forces were about 50% greater than the British, indicating an $S_{ss}$ of approximately 3 million. $S(0)$ is the size of the French forces in theatre at the start of the war (British forces were negligibly small) and they totalled approximately 2 million.[5] The value of $(k + l)$ is known also from Voevodsky ($1/1.3 = 0.77$). What remains to be calculated is $C(\tau)$ – the total French and British permanent losses (as defined above) for the first 16 months of the war. This comes to approximately 1.6 million men.[6] The value of $k$ comes out as 0.32 (and $l = 0.45$), making $k/l = 0.71$. The steady-state war requirement that, here, $(k/l)^2 < 1$, is met.

## War Termination and Strategic Choices

The evidence suggests that tendencies towards steady-state have been present in most of the major wars of the past century and more. The deliberately simple mathematical model presented shows how steady-state war can arise. But wars end: they are sometimes still won and lost. Why should this be so?

There are two broad answers. Wars can terminate *because of* their steady-state tendencies, or they can be terminated, as a result of strategic choices, *despite* their steady-state tendencies.

In the former category, long wars can

[5] This is the most problematic data item. According to Churchill (1927: 23), the French began the war with 1.3 million men actually under arms. As soon as the war started, a further 1.2 million reservists immediately joined the colours but were obviously not all ready for active service straight away. Accordingly, we have taken the figure for the size of the French forces at the start of the war as the average of 1.3 and (1.3 + 1.2) or 2 million in round figures.

[6] Churchill (1927: 52) gives separate total figures for British and French killed, missing, taken prisoner and wounded for the war on the Western Front up to January 1916. Wounded alone make up approximately three-quarters of the British total and two-thirds of the French. Of the wounded, approximately one-third played no further part in the war and are added in as further permanent losses (following Churchill's own rule of thumb on p. 56). The appallingly high total of 1.2 million dead or seriously wounded in 16 months is mainly French, and arises from the excess of élan they allowed themselves in the early days of the war.

easily correspond to a 'mutually (if not necessarily equally) hurting stalemate' where duration leads to settlement. In rational-choice language,[7] the participants may be presumed to compare costs and benefits when deciding whether to continue the war. The present model suggests that the incremental cost of a further period of war (e.g. one more year) which has reached the steady-state will be constant ($E$). The reason is that in or near the steady-state, human casualties accumulate linearly with time as do the material costs of inflicting casualties on the other side (assuming a fixed financial cost per casualty inflicted).[8] The incremental expected benefit of continuing the war for a further period will depend upon the probability that the war will end in that period and on the value put on a settlement being reached. If, as a simplification, we consider a steady-state war to have a fixed but normally small probability, $p$, of ending in any given 12-month period, the mean (expected) length of the war will be $1/p$ years. Thus the incremental expected benefit of continuing the war for any one side will depend linearly on the value of a settlement ($V$) times the probability of a settlement, $p$, in the coming year. The incremental expected cost will depend linearly on $(1 - p)$, the probability that the war will *not* terminate in the upcoming period, times $E$, the cost. The difference between the incremental expected benefit and the incremental expected cost will eventually become negative as the valuation put on $p$ approaches zero. Where a war has already lasted $Y$ years, $p$ (or, strictly, its most probable value) must be less than or equal to $1/Y$, so as $Y$ increases $p$ does approach zero, and cost–benefit calculations, once the war has been going for long enough, will point to the logic of terminating the war for the side with the smaller ratio of $V$ to $E$.

Comparable considerations throw light on the beginning of wars. If the tendency of war is towards stalemate, it might be wondered, as foreshadowed above, why wars are embarked upon in the first place. Indeed another way of asking the same question is to note that a long war or a war which, as Clausewitz says, involves 'much waiting' is to the benefit of the defending side, not the attacker, so why should anyone initiate it? (Clausewitz, 1834/1984) But Wagner rightly points out that in such a war, fighting might still be preferred when each side is optimistic about the effect of stalemate on the willingness of the other side to compromise (Wagner, 1994: 603). And the analysis in the preceding paragraph indicates that adverse cost–benefit calculations will generally present themselves to one side first. Mack's comment on the relatively weak position in a stalemated war of a country with limited goals and low cost tolerance (especially, one might add, to casualties), written in the light of US experience in Vietnam (Mack, 1975), is obviously applicable here.

In explaining the latter category of 'termination despite steady-state tendencies', it is important to realize that wars may end because the preconditions for steady-state tendencies to be maintained may themselves cease to be satisfied. These preconditions can, as we have seen, normally be telescoped into the single requirement that $kq$ should be less than $lp$, with $k/l$ and $q/p$ both small (i.e. $< 1$). But in full the preconditions are threefold (see Appendix). First, $kq/lp$ should be less than 1, as above. Second, $R_m/B_m$ should

[7] For the locus classicus of the impact of cost–benefit calculations on war termination but with an emphasis on negotiation aspects, see Pillar (1983).

[8] In the steady-state, force levels in theatre have ceased rising. The rate at which casualties (killed and seriously wounded) are being experienced must therefore equal the rate at which fresh forces are introduced. The latter rate is constant since in this model it is proportional to the difference between the (fixed) maximum level of forces that can be supported in the field and the existing (fixed) steady-state figure.

be greater than $k/l$ and third $p/q$ should be greater than $R_m/B_m$.

The war in the Falklands/Malvinas in 1982 and the war in the Gulf in 1990–91 were not steady-state wars; both were short. In the former case, the capacity on both sides to resupply was very restricted, partly by geographical considerations. In the latter case, the capacity of the Iraqis to resupply was severely and deliberately diminished partly by the arms embargo and partly by the largely unopposed intensive bombing campaign by the US-led coalition that preceded the ground war. In other words the condition for a steady-state war that $k/l$ and $q/p$ should be small may simply not have been met, since the supply terms $l$ and $p$ may have been or may have been made rather small.

Second, the Gulf war provides a more general clue. In a steady-state war, the very fact that it is tending towards a stable equilibrium implies that some effort can be diverted from the front-line with comparative impunity. If this effort in turn can be channelled into weakening the enemy's supply infrastructure, the military situation can be turned from a stable one or potentially stable one into an unstable one in favour of the side which is the more successful, relatively, at undermining the enemy's supply infrastructure.[9] Note that this infrastructure includes not only lines of supply, but also the sources of supply within the enemy's economy and society. It is a characteristic of modern warfare that it is often possible, at least in principle, to disrupt the enemy's supply infrastructure before defeating the enemy in the field (i.e. whilst the situation in the field is a stable one) by virtue of the existence of air and missile power.[10]

Taking advantage of the stable situation in the field to divert effort towards undermining the enemy's supply network can mean one of two types of warlike activity that are distinguishable in theory if not, possibly, always in practice.

From Blue's perspective, effort can be directed either at reducing the size of the enemy's '$l$' (the logistics term), or (see Appendix) the size of $R_m$, the maximum force Red can support (or is prepared to support) in the field. Making attacks behind the lines on Red's logistic or military transport facilities, from the air or where appropriate from the sea (as with the British interdiction of Argentine reinforcements in the Falklands), would reduce both $l$ and $R_m$, as it would push downwards both the rate at which Red could move fresh troops into action and its capacity to supply troops already in place with munitions, fuel, food, etc. Equally, non-military activity could have military effects. Diplomatic manoeuvring to reduce Red's capacity to attract or retain allies will certainly affect $R_m$. And even $l$ could be reduced by persuading strategically placed neutrals to deny Red transit rights. Strategic bombing, which might be described as attacks on economic targets of all kinds, would in the end reduce $R_m$ to the extent it put economic growth into reverse, and have a damaging effect on $l$.

So, one way in which steady-state wars may be won or lost is through greater success by one side than the other in altering the values of the key parameters of the war as indicated by the model. The importance of this point cannot be overstressed. *Quantitative* change here can have *qualitative* effects. Comparatively small changes

[9] A stable situation in the field could be taken advantage of in a different way. As an alternative to diverting resources in order to reduce Red's $l$ and $R_m$, they could be diverted internally towards increasing Blue's own '$k$'. This would translate into the research and development of new and more destructive battlefield weapons.

[10] This is not to deny that some capacity to disrupt supply was present in earlier times. One army could manouevre around another and strike at the latter's depots, for example.

in the values of the key parameters achieved through the diversion of armed force to off-battlefield targets – something that can be done, to repeat, with comparative impunity in the context of a steady-state war – can bring about very large consequences through altering the character of the conflict from one of stability to one of instability. Ironically, the opportunity to do this is often presented in the first place by the stable condition of the war.

It will be apparent that this is tantamount to saying that, in different ways, wars are won not so much on the battlefield where they have tendencies to stalemate, but off the battlefield.

The extreme manifestation of this view comes with a war in which the battlefield element becomes almost irrelevant. Where there is a one-sided advantage in the applicability of force off the battlefield, even if on the battlefield there is a potential for stalemate, the stalemate can be pre-empted by initiating the war off the battlefield and so damaging the opponent's infrastructure (diminishing $l$ and $R_m$) as to make the battlefield phase a formality. This formality (as it transpired to be, for the winners) may still be gone through, as in the Gulf War of 1990–91, or defeat may be conceded during the off-battlefield phase, in anticipation of the battlefield consequences of failure to concede, as, arguably, might have been the case in the Kosovo air-only campaign of 1999.

The tendency to stalemate is undiminished when wars are fought essentially on the battlefield, either (in a land war) because of the relative physical absence of airpower due to the comparatively primitive nature of the combatants, or because of the relative inutility of airpower where the other side has very short lines of supply, as in guerrilla war, or because airpower is available but used restrictedly because of political decisions that the war should be limited to the involvement of targets on and near the battlefield only (with this limitation enforced by mutual intrawar deterrence).

Countries aware of a generalized intolerance of stalemated war will avoid it, either by avoiding war altogether or by aiming to see to it that any wars they do participate in are not stalemated (i.e. are short rather than long). Of course from the particular perspective of the present article this means at the least an extensive pre-war arsenal of long-range weapons for use off the battlefield against the enemy's supply lines and supply base. It involves therefore high peacetime defence spending (a short war must be fought with weapons in being and therefore must be paid for in advance of the event).

## Conclusion

This article presents a deliberately simple but general mathematical model of war, with an empirical basis in the pioneering and unaccountably neglected work of Voevodsky. The model is Lanchester in form, but aims for explicitness concerning the meaning of the parameters involved and pays special attention to solutions that correspond to steady-state outcomes for the balance of forces engaged in theatre.

Correspondingly, the model is used to explain the long duration of a number of major wars in the past century and a half, and therefore has something to say about the question of war termination. The model makes it clear how wars can terminate *because of* their steady-state tendencies, or how they can be terminated, as a result of strategic choices, *despite* their steady-state tendencies. In the former case, attention is paid to the interface between the model and rational choice theories of war termination. In the exploration of the latter case, what the model does is present an explanation for the shifts in military strategy in the present century towards waging war off the battlefield as well as on it, with the culmination of these shifts being seen, arguably, within the past decade, in the Gulf War of 1990–91 and the Kosovo War of 1999, where the battlefield employment of force was in one case a formality and in the other unnecessary.

## References

Brown, David & Peter Rothery, 1993. *Models in Biology.* Chichester: Wiley.

Churchill, Winston S., 1927. *The World Crisis 1916–1918: Part 1.* London: Thornton Butterworth.

Clausewitz, C. von, 1834/1984. 'On War', trans. and ed. by Michael Howard & Peter Paret, Princeton, NJ: Princeton University Press.

Mack, Andrew, 1975. 'Why Big Nations Lose Small Wars', *World Politics* 27(2): 175–200.

Mason, T. David & Patrick J. Flett, 1996. 'How Civil Wars End: A Rational Choice Approach', *Journal of Conflict Resolution* 40(4): 546–568.

Massoud, Tansa George, 1996. 'Review Essay: War Termination', *Journal of Peace Research* 33(4): 491–496.

May, Robert M., 1976. 'Simple Mathematical Models with Very Complicated Dynamics', *Nature* 261: 459–467.

May, Robert M., 1981. 'Models for Two Interacting Populations', in Robert M. May, ed., *Theoretical Ecology: Principles and Applications.* Oxford: Blackwell Scientific (78–104).

Morse, Philip M. & George E. Kimball, 1962. *Methods of Operations Research.* Cambridge, MA: MIT Press.

Pillar, Paul R., 1983. *Negotiating Peace: War Termination as a Bargaining Process.* Princeton, NJ: Princeton University Press.

Smith, J. Maynard, 1968. *Mathematical Ideas in Biology.* Cambridge: Cambridge University Press.

Voevodsky, John, 1971. 'Modeling the Dynamics of Warfare', in Douglas E. Knight, Huntington W. Curtis & Lawrence J. Fogel, eds, *Cybernetics, Simulation and Conflict Resolution.* New York & Washington, DC: Spartan (145–170).

Wagner, R. Harrison, 1994. 'Peace, War, and the Balance of Power', *American Political Science Review* 88(3): 593–607.

Zartman, I. William, 1985. *Ripe for Resolution.* Oxford & New York: Oxford University Press.

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## Appendix

### *System Stability*

$$dr/dt = lR_m - lr - kb \quad (1)$$

$$db/dt = pB_m - pb - qr \quad (2)$$

From Equations (1) and (2) a steady-state outcome requires not only that dr/dt = db/dt = 0, but also that the relationship of forces at this juncture is a stable one. That is to say, an equilibrium is reached which is robust and not easily upset by increases or decreases in the size of the opposing forces arising as a result in the latter case, for example of Clausewitz's 'friction'.

To explore system stability, the first step is to solve the equations for $r$ and $b$, given that $dr/dt = db/dt = 0$. Simple algebra shows that:

$$r_e = (R_m l/k - B_m)/(l/k - q/p) \quad (3)$$

$$b_e = (B_m p/q - R_m)/(p/q - k/l) \quad (4)$$

These values obviously correspond to some sort of equilibrium situation in that the size of both Red's and Blue's forces are now steady. But the equilibrium might however be unstable and we are interested in the circumstances where homeostasis applies, i.e. where fluctuations in the size of the armies of one or other side about the equilibrium point should be damped out.

When $dr/dt = db/dt = 0$, we can re-write Equations (1) and (2) as:

$$r_e = R_m - (k/l)b_e \quad (5)$$

and

$$b_e = B_m - (q/p)r_e \quad (6)$$

How would $r_e$, the equilibrium value of Red's forces, be affected (in Equation (5) by an upward shift, $\Delta b_1$, in the size of Blue's forces?) It would be pushed downwards, according to Equation (5), by an amount $(k/l)\Delta b_1$. And how would this fall in Red's forces impact on the size of Blue's equilibrium strength? By Equation (6) a fall in Red's equilibrium strength of $\Delta r_1$, leads to an increase in Blue's strength of $(q/p)\Delta r_1$. If we call this new upward increment to Blue's strength $\Delta b_2$, its value is:

$$(q/p) \times (k/l)\Delta b_1.$$

For homeostasis to apply, that is to say for the equilibrium described to be stable, $\Delta b_2$ should be less than $\Delta b_1$, or, $qk/pl < 1$. If this condition is met, disturbances at the equilibrium point dampen down, with a new equilibrium established not far from the original point (an analysis beginning instead with a shift in the size of Red's equilibrium forces would lead to the same conclusion). When this condition is not met, disturbances quickly become amplified and equilibrium is lost.

Applying this condition to Equations (3) and (4), since $r_e$ and $b_e$ must themselves be positive, it is also necessary for stable equilibrium that $R_m/B_m > k/l$ and that $p/q > R_m/B_m$.

All these conditions should be satisfied without much difficulty (May, 1981: 88), provided $k/l$ and $q/p$ are both small (less than 1). This was the position taken in the main text. Our World War I calculations showed that $k/l$ (and $q/p$, which was taken to be the same) was approximately 0.71 in that war.

Of course, there must be limits to the homeostatic process. A sudden drop in Blue's force size which exceeded $b_e$ could not be recovered from. If such a drop were inflicted by a sudden increase in Red's forces, due, say, to the acquisition by Red of a new ally, this would imply that an addition to Red's strength of $b_e \times p/q$ or more could not be recovered from by Blue on its own. To be sure, this will normally correspond to a very large increment in Red's strength. Second, the smaller the value of $k/l$ and $q/p$ the stronger is the homeostatic tendency. On the other hand, the larger these two stability parameters are (the nearer they are to unity) the longer will be the time needed for a situation of equilibrium to be re-established after it had been disturbed in some way. And the longer this period of time is, the more opportunity there will be for other (possibly chance) influences to interfere and determine outcomes.