Test 1, Feb 6 2019, Intro Advanced Math.

1. Let A, B, C be sets. If $A \subseteq B \bigcup C$ then show $A - B \subseteq C$.

2. Let p, q be statements. Which of the following statements are logically equivalent, if any? Which are tautologies, if any?

   $S_1 :\quad p \vee q$
   $S_2 :\quad (\neg q) \vee (p \Longrightarrow q)$
   $S_3 :\quad (\neg q) \Longrightarrow p.$

3. Give the definitions of:

   (a) A function $f : A \to B$ is one-to-one (injective) when:

   (b) The function $f^{-1} : P(B) \to P(A)$ is defined as follows: If $T \in P(B)$ then $x \in f^{-1}(T)$ if and only if:

   (c) If $S \subseteq L$ where L is a p.o.set with ordering $\leq$ then u is a greatest lower bound for S when:

   (d) If u is a bottom element of S, must u then also be a greatest lower bound for S? (Yes/No with brief explanation).

4. Let $f : A \to B$ and $g : B \to C$. If the composition $g \circ f : A \to C$ is onto then show that g is onto.

5. Suppose L is a chain and that $S \subseteq L$ has no top element.
   Then show $\forall_{a \in S} \exists_{b \in S}\ b > a$.

Writing proofs.

1. **Direct proof for $p \implies q$.**
   Assume: p. To prove: q.

2. **Proving $p \implies q$ by contrapositive.**
   Assume: $\neg q$. To prove: $\neg p$.

3. **Proving S by contradiction.**
   Assume: $\neg S$. To prove: a contradiction.

4. **Proving $p \implies q$ by contradiction.**
   Assume: p and $\neg q$. To prove: a contradiction.

5. **Direct proof for a $\forall_{x \in A} P(x)$ statement.**
   To ensure you prove $P(x)$ for *all* (rather than for *some*) x in A, do this:
   Start your proof with: Let $x \in A$. To prove: $P(x)$.

6. **Direct proof for $\exists_{x \in A} P(x)$ statement.**
   Take $x :=$ [write down an expression that is in A, and satisfies $P(x)$].

7. **Proving $\forall_{x \in A} P(x)$ by contradiction.**
   Assume: $x \in A$ and $\neg P(x)$. To prove: a contradiction.

8. **Proving $\exists_{x \in A} P(x)$ by contradiction.**
   Assume: $\neg P(x)$ for every $x \in A$. To prove: a contradiction.

9. **Proving S by cases.**
   Suppose for example a statement p can help to prove S. Write two proofs:
   Case 1: Assume p. To prove: S.
   Case 2: Assume $\neg p$. To prove S.

10. **Proving $p \wedge q$**
    Write two separate proofs: To prove: p. To prove: q.

11. **Proving $p \iff q$**
    Write two proofs. To prove: $p \implies q$ To prove: $q \implies p$.

12. **Proving $p \vee q$**
    Method (1): Assume $\neg p$. To prove: q.
    Method (2): Assume $\neg q$. To prove: p.
    Method (3): Assume $\neg p$ and $\neg q$. To prove: a contradiction.

13. **Using $p \vee q$ to prove another statement r.**
    Write two proofs:
    Assume p. To prove r.
    Assume q. To prove r.

14. **How to use a for-all statement $\forall_{x \in A} P(x)$.**
    You need to produce an element of A, then use P for that element.