Test 1, Intro Advanced Math

1. Let A, B, C be sets. Assume $C - A \subseteq B$, then show $C - B \subseteq A$.

2. Let p, q be statements. Which of the following statements are logically equivalent, if any? Which are tautologies, if any?

   $S_1 : \quad p \vee (p \implies q)$
   $S_2 : \quad p \vee (q \implies p)$
   $S_3 : \quad p \implies q$
   $S_4 : \quad (\neg p) \implies (\neg q)$.

3. Give the definitions of:

   (a) A function $f : A \to B$ is onto when:

   (b) L is partially ordered set when:

   (c) L is a chain when:

   (d) If $S \subseteq L$ then a lower bound of S is:

   (e) If $S \subseteq L$ then a bottom element of S is:

4. If $f : A \to B$ is onto and $g : B \to C$ is onto, then show that the composition $g \circ f : A \to C$ is onto.

5. Suppose L is a partially ordered set but not a chain. Show that there is a non-empty set $S \subseteq L$ that has no bottom element.

6. Give an example of a partially ordered set L and a non-empty subset $S \subseteq L$ where S has a greatest lower bound but not a bottom element.