Multiplying polynomials.

A polynomial f in Q[x] is an expression:

f = sum a.i * x^i

where a.i are rational numbers. For example:

```
> f := 3/2 * x^3 + 5*x - 3;
```

$$f := \frac{3}{2}x^3 + 5x - 3$$

```
> degree(f,x); # = highest power of x
```

$$3$$

```
> ldegree(f,x); # = lowest power of x. Note: 3 = 3*x^0 so lowest
  power = 0
```

$$0$$

```
> list_of_coefficients := [seq( coeff(f,x,i), i=0..degree(f,x) )];
```

$$list\_of\_coefficients := \left[-3, 5, 0, \frac{3}{2} \right]$$

Now we will represent polynomials in the following way:

The list L = [a0, a1, a2, ..., an] will correspond to the polynomial
f = an * x^n + ... + a1*x^1 + a0*x^0 (note: x^1 is just x and a0*x^0 is just a0).

Assignment 1: Write a procedure called:

add_poly := proc(L1, L2)
local ...
...
end:

whose input is two lists, and whose output is one list that corresponds to the sum. Note: Maple can already add lists:

```
> [3,6,-1,3/4] + [0,0,3,1];
```

$$\left[3, 6, 2, \frac{7}{4} \right]$$

```
> [1,2,3,4] + [1,2,3];
Error, adding lists of different length
```

but the addition only works if the lengths are the same.

So your procedure should do the following:
if the lengths are the same (nops(L1) =nops(L2)) then just use +
if the lengths are not the same, then make the shorter list longer by adding 0's at the end, and then add the lists.

At the end, if the last entry happens to be 0, for example when you add:
f1 := 3*x^2 - 1;
f2 := -3*x^2 + 5*x + 1;
then you have L1 = [-1,0,3] and L2 = [1,5,-3] and the sum is [0,5,0] but f1+f2 = 5*x which has only degree 1, so you want to simplify the list L1+L2=[0,5,0] by removing the last 0. So if the input is L1, L2, you compute first L1+L2=[0,5,0] and then you remove the last 0. You can not do that by:
subs(0 = NULL, ...)
because that would also remove the first 0 which is not what we want.

So at the end, if L = L1+L2 and f = f1+f2 then you want: nops(L) = degree(f1,x) + 1.

Assignment 2:

Note that in Maple you can multiply a list by a number:

```
> -2 * [3,45,0,-2];
```

[-6, -90, 0, 4]

So if a polynomial f is represented by a list L, and if k is some number, then this will let you determine the list of coefficients of k*f.

The next step is the determine the list of: f, x*f, x^2*f, ... etc.
Note that to compute the list of x*f, you need to add one entry (with value 0) at the beginning of the list.

Now, if g = b0*x^0 + ... + bn*x^n then you can calculate g*f as:
b0*f + b1 * x*f + b2 * x^2*f + ...
and you can compute this list by using the add_poly command from assignment 1.

This way you can write a procedure:
mult_poly := proc(L1, L2)
...
end:
that computes the product of two polynomials.

Assignment 3:

If L is a list of numbers, and k is a number, and you do: k*L then the number of multiplications number*number that Maple must do equals: nops(L), there is one multiplication for each element of L.
If L1 and L2 both have n entries, then how many number*number multiplications will be done during

the computation: mult_poly(L1, L2) ?