Linear algebra, test 1, Sep 24 2018, 2:30–3:15

1. (15 points). Let

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} a \\ a \\ 1 \end{pmatrix}.$$

For which value(s) of a would w be an element of $\mathrm{SPAN}(v_1, v_2)$?

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ send $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ to $\begin{pmatrix} x_1 + x_2 \\ x_2 \\ 2x_1 + 3x_2 \end{pmatrix}$.

(a) (15 points). Give the matrix of T.

(b) (5 points). Give the definition of injective (one-to-one).

(c) (5 points). Give the definition of surjective (onto).

(d) (5 points). Is T injective?

(e) (5 points). Is T surjective?

3. Let

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 & 1 \\ -1 & 1 & -1 & 2 & 1 \\ 0 & -1 & -1 & -1 & -2 \end{pmatrix}.$$

(a) (10 points). Compute the rref (*reduced* row echelon form) of matrix A. Indicate which row-operations you used (e.g. $R_2 \leftarrow R_2 + R_1$).

(b) (10 points). The matrix A is the augmented matrix of the following system of equations:

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ -1 & 1 & -1 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}.$$

Give *all* solutions for this system. Write the solutions in *parametric vector form.*

Note: don't do any more row-reduction for this question, you already did that in part (a).

(c) (10 points). Let $v_1, \ldots, v_5$ be the columns of A. Answer the following questions. Note: don't do any computation for this question, you already did that in part (a).

i. Is v_2 in SPAN(v_1)? Yes/no. If yes, then write v_2 as a linear combination of v_1.

ii. Is v_3 in SPAN(v_1, v_2)? Yes/no. If yes, then write v_3 as a linear combination of v_1, v_2.

iii. Is v_4 in SPAN(v_1, v_2, v_3)? Yes/no. If yes, then write v_4 as a linear combination of v_1, v_2, v_3.

iv. Is v_5 in SPAN(v_1, v_2, v_3, v_4)? Yes/no. If yes, then write v_5 as a linear combination of v_1, v_2, v_3, v_4.

4. (a) (5 points). Give the definition: Vectors v_1, v_2, v_3 are linearly *dependent* when:

   (b) (8 points). If v_1, v_2 are linearly dependent, must v_1, v_2, v_3 then also be linearly dependent? Yes/no. Explain using the definition from part (a).

   (c) (7 points). If A is a matrix and x is a vector, must Ax be a linear combination of the columns of A? Explain.