Linear algebra, test 3.

March 25, 2004

1. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \end{pmatrix}$$

   (a) (5 points). Compute the reduced row echelon form of A.

   (b) (8 points). Write down a basis B of $\mathrm{Col}(A)$.

   (c) (12 points). Give a basis C of $\mathrm{Nul}(A)$.

   (d) (10 points). Let v_1, v_2, v_3, v_4 be the columns of matrix A. What are: $[v_1]_B$, $[v_2]_B$, $[v_3]_B$ and $[v_4]_B$.

   (e) (10 points). Let

$$w = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

   What's the easiest way to tell if w is in $\mathrm{Nul}(A)$ or not?
   If w is in $\mathrm{Nul}(A)$, then compute its coordinate vector $[w]_C$ with respect to the basis C you computed in question (c).

2. Let $u = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. Let $V = \mathrm{SPAN}\{u, v\}$ and $B = \{u, v\}$ a basis of V.

(a) (5 points). If w is some vector in V for which $[w]_B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ then what is w?

(b) (15 points). Which of the following vectors are in V? For each that is in V, give the coordinate vector with respect to B.

$$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Question 2 continues on the next page.

(c) (3 points). Give a matrix A for which: $V = \mathrm{Col}(A)$.

(d) (12 points). Give a matrix N for which: $V = \mathrm{Nul}(N)$.

(e) (2 points). True or false: The vectors in question (b) form a spanning set for V?

(f) (2 points). True or false: If we take those vectors in question (b) that were in V, then we get a spanning set for V?

(g) (2 points). True or false: If we take those vectors in question (b) that were in V, then we get a basis for V?

3. (10 points). For each of the following, mention if it is a vector space or not.
If it is not a vector space, then say why it is not a vector space by writing one of the following three things: "does not contain the zero vector", or "is not closed under addition", or "is not closed under scalar multiplication".
If it is a vector space, then no further explanation will be necessary.

   (a) $V = \{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\}$.

   (b) $V = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid ab = 0\}$.

   (c) $V = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid a + b + 3 = 0\}$.

   (d) $V = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid 2a - 5b = 0\}$.

   (e) $V = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid a \geq b\}$.

4. (5 points). If $B = \{t^2 + t,\ t + 1,\ 1\}$ then what is $[t^2 - t + 1]_B$?