Linear algebra, test 2, Feb 26 2004.

1. (15 points). Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_3 \\ x_1 \end{pmatrix}$$

   Give the matrix of T.

2. Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -2 \\ 0 & 1 & -1 \end{pmatrix}$$

(a) (10 points). Compute the determinant of A.

(b) (10 points). Compute the inverse of A.

(c) (5 points). Use your answer of part (b) to solve:

$$AX = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

3. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be a 180^o rotation around the origin.
So $S(v) = -v$ for every $v \in \mathbb{R}^2$.

   (a) (5 points). Let A be the matrix of S. Compute A.

   (b) (5 points). Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map "rotation around the origin with an angle of 90^o counter-clockwise". Let B be the matrix of T. Compute B.

   (c) (3 points). Explain without computing B^2 why $B^2 = A$.

   (d) (3 points). Explain without computing why $B^3 = B^{-1}$.

4. (20 points). Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map and suppose that:

$$T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

and

$$T \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Give the matrix of T. What is the inverse of this matrix?

5. (24 points). True or false?

(a) If $AB = AC$ and if $B \neq C$ then A can not be invertible.

(b) If A, B are square matrices and AB is the identity matrix then BA is also the identity matrix.

(c) If T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^5$ then T is never one-to-one.

(d) If T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^5$ then T is never onto.

(e) If A is a square matrix, and $B = A^T$ is the transpose of A, and if B can not be row-reduced to the identity matrix then $AX = 0$ must have a non-trivial solution X.

(f) If A can be row-reduced to B then there exists an invertible matrix C such that $B = CA$.

(g) If A and B are square matrices, both not zero, then AB is also not zero.

(h) If A is a square matrix and $\det(A) = 0$ then $AX = 0$ has only the trivial solution $X = 0$.

(i) If A and B are square matrices, and if AB is invertible, then BA must also be invertible.

(j) If A has more rows than columns, then the columns of A can not be linearly independent.

(k) If A is a m by n matrix and the reduced row echelon form has a zero row then $AX = 0$ has a non-trivial solution X.

(l) If A can be row-reduced to the identity matrix then A^T (the transpose of A) can also be row-reduced to the identity matrix.