1. A linear map $T : \mathbf{R}^n \to \mathbf{R}^m$ is always given by matrix multiplication. That means, if T is a linear map from $\mathbf{R}^n$ to $\mathbf{R}^m$ then there is a matrix A such that if $v \in \mathbf{R}^n$ then the following will have the same result:
$*$) applying T to v (result is: $T(v)$).
$*$) Multiplying v on the left by A (result is: Av).
How are we going to find this matrix A for which $T(v) = Av$ for all v?
Answer: Take the *standard basis* (see below) and apply T to all vectors in this standard basis (in some exercises you may need to use the idea in item 12 below). The resulting vectors then form the columns of A.

2. The standard basis of $\mathbf{R}^n$
$=$ the columns of the n by n identity matrix.

3. The identity matrix I is a square matrix with 1's on the diagonal and 0's elsewhere. Multiplying by I does nothing (hence the name: identity matrix). So $Iv = v$ for all vectors v. Moreover, $IA = A$ for all matrices A, and also $AI = A$ for all A.

4. The rank of T is the number of pivots in the rref of the matrix A that belongs to T.

5. If $T : \mathbf{R}^n \to \mathbf{R}^m$ then T is one-to-one if the rank is n.

6. If $T : \mathbf{R}^n \to \mathbf{R}^m$ then T is onto if the rank is m.

7. T is invertible if it is one-to-one and onto.

8. If $T : \mathbf{R}^n \to \mathbf{R}^m$, and if $n \neq m$ then T is not invertible.

9. If $T : \mathbf{R}^n \to \mathbf{R}^m$, and if $n = m$ then T is invertible when the rank of the corresponding matrix is n. But then the rref of that matrix must be I.

10. $T : \mathbf{R}^n \to \mathbf{R}^m$ is invertible if and only if the corresponding matrix is square (so $m = n$) *and* can be row-reduced to I.

11. To compute the inverse of a matrix A, rowreduce $(A\ I)$ to $(I\ \ldots)$. If this row-reduction is impossible (if A can't be row-reduced to I) then there is no inverse of A. But if the row-reduction is possible, you'll find A^{-1}, because then $(A\ I)$ row-reduces to $(I\ A^{-1})$.
For a 2 by 2 matrix there is a short formula for the inverse (see the book).

12. If $T(v_1)$ and $T(v_2)$ are known, then you can compute $T(w)$ for any linear combination w of v_1, v_2. How? First, write w as a linear combination of v_1, v_2, that is, find c_1, c_2 such that $w = c_1v_1 + c_2v_2$. We remember how to find those c_1, c_2, by row-reducing $(v_1v_2\ w)$. Then, once we know these c_1, c_2 for which $w = c_1v_1 + c_2v_2$ then we can compute $T(w) = c_1T(v_1) + c_2T(v_2)$.

13. If we want to do item 12 for several w's, say: w_1, w_2, w_3 then we would have to solve three systems, with these augmented matrices:
$(v_1v_2\ w_1)$, $(v_1v_2\ w_2)$ and $(v_1v_2\ w_3)$.
We can solve these three systems simultaneously by row-reducing just this augmented matrix: $(v_1v_2\ w_1w_2w_3)$ (so the right-hand side in this augmented matrix has 3 columns).

14. The idea in item 12 works also if we have more than just two vectors v_1, v_2. If we have k vectors $v_1, \ldots, v_k$, and if we know $T(v_1), \ldots, T(v_k)$ and we want to compute $T(w)$ for some linear combination w of $v_1, \ldots, v_k$, then the first step would be to row-reduce $(v_1 \cdots v_k\quad w)$, and everything works in the same way.

15. Section 2.1. Memorize theorems 2, 3.
Section 2.2. It is useful to memorize the formula in theorem 4 (if you forget this formula, then you're not yet lost though, because you can then fall back on the method given in item 11).
Memorize theorems 5, 6, 7.
Section 2.3. Memorize all of the items in theorem 8. In the test, I might pick any two out of all of those statements, and ask you if the first one is true whether the second one is then also true. If you know that the matrix is square, then the answer to that question would be yes. No explanation will be required, just memorize that these statements in theorem 8 are all equivalent *provided that you already know that A is square!!* (so: if you don't know that A is square, then don't use anything from theorem 8 because then it may be wrong).