1. elementary row operation: See book.
   REF (row echelon form): See book.
   RREF (reduced row echelon form): See book.
   rhs = right hand side

2. Inconsistent row: A row where the coefficient-side is all 0 while the rhs in this row is not 0.

3. System is consistent: when the REF (or RREF) does not contain an inconsistent row.

4. System is inconsistent: when the REF (or RREF) does have an inconsistent row. If you encounter any inconsistent row during row-reduction, then the system is inconsistent.

5. If the rhs is completely zero, we can never get an inconsistent row, so the system is automatically consistent. Note that during row-reduction in class, I'll often omit the rhs if the rhs is completely zero. So if the rhs is zero, I'll compute with just the coefficient matrix, not the augmented matrix.

6. Augmented matrix = a matrix that contains the coefficient matrix and also the rhs.

7. Rank of the coefficient matrix = the number of non-zero rows in the REF (or RREF) of the coefficient matrix.

8. Free variables: You have free variable(s) if in the coefficient side of the REF (or RREF) there are column(s) without a row-leader. Note that having free variables has nothing to do with having a zero-row (except when the coefficient matrix has the same number of rows as columns, see the explanation in item 15c below). Free variable(s) only has to do with having column(s) on the coefficient side of the REF without a row-leader.

9. Basic variable = not a free variable. A variable is basic if the corresponding column in the REF (or RREF) has a row-leader (or pivot).

10. Rank of the coefficient matrix
    = # non-zero rows in the REF (or RREF) of the coefficient matrix.
    = # row-leaders in the REF of the coefficient matrix.
    = # in the REF of the coefficient matrix that have a row-leader.
    = # basic variables.

11. Rank of the coefficient matrix is at most # variables (remember that the # variables = # columns of the coefficient matrix).

12. If the rank of coefficient matrix = # variables then: no free variables.

13. Rank of the coefficient matrix is at most # equations (remember that # equations = # rows).

14. If the rank equals # rows then there are no zero-rows in the coefficient matrix which means (see "Inconsistent row" in item 2) that the system is consistent for any rhs.

15. (a) System consistent for any rhs when: rank(coefficient matrix) = # rows.

    (b) No free variables when: rank(coefficient matrix) = # columns.

    (c) Notice that if # rows = # columns in the coefficient matrix, then we're in a very special situation, because then (a) is true if and only if (b) is true.
    So if # equations = # variables, then we're in a very special situation, namely you get zero-row(s) in the coefficient side if and only if you get free variable(s). But in general (if we don't know if the number of equations is the same as the number of variables) then zero-rows and free variables have nothing to do with each other. Remember that for the true/false questions on the next tests, but actually, you need to know that for the other questions too because if you don't, you'll get the question wrong each time I give you a tricky question.

16. m by n matrix. Means m rows, n columns.

17. m by n system. Means m equations in n variables.

18. If A is a matrix and x is in $\mathbf{R}^n$ (this means that x has n entries) then Ax is defined when the number of entries (the number of rows) of x equals the number of columns of A.

19. If Ax is defined, and if $x_1, x_2, \ldots, x_n$ are the entries of x, then $Ax = \mathrm{Col}_1(A)x_1 + \cdots + \mathrm{Col}_n(A)x_n$.
    Note that if $x_1, \ldots, x_n$ are unknown, then the equation $Ax = b$ is the same as the system of equations whose augmented matrix is $(A\ b)$, so A is coefficient matrix and b is the rhs.

20. The system $Ax = b$ is consistent if and only if b is a linear combination of the columns of A.

21. Writing w as a linear combination of v_1, v_2, v_3 means this: compute numbers c_1, c_2, c_3 and then write $w = c_1v_1 + c_2v_2 + c_3v_3$.
How can you find these numbers? Well, sometimes it's easy, if you happen to notice that say $w = v_1 - v_3$ then you have $c_1 = 1, c_2 = 0, c_3 = -1$. If you don't happen to notice something like that, then you have to solve this augmented system: $(v_1\ v_2\ v_3\quad w)$. If it is consistent, then w is a linear combination of v_1, v_2, v_3, and then the solution of this system will give you the numbers c_1, c_2, c_3.

22. Vectors $v_1, \dots, v_n$ are linearly dependent if:
There exist numbers $c_1, \dots, c_n$, not all 0, such that $c_1v_1 + \cdots c_nv_n = 0$ (this last equation is then called a linear relation between $v_1, \dots, v_n$).
How are you going to find such $c_1, \dots, c_n$ if they exist?
Answer: Just write $A = (v_1\ v_2\ \cdots\ v_n)$ and then solve $Ax = 0$. Any non-zero solution (meaning that at least one of the n numbers is not zero) will give you the $c_1, \dots, c_n$ of a linear relation.

23. A non-trivial solution of $Ax = 0$ is a solution that is not entirely zero.
Such a solution exists if and only if the columns of A are linearly dependent.
Such a solution exists if and only if there are free variable(s). Now read up on free variables.