```
> restart; read "/Users/hoeij/Downloads/Reduce_Order_Algorithm.txt" :
> with(LREtools) :
  _Env_LRE_tau := tau;
  _Env_LRE_x := x;
```

$$\_Env\_LRE\_tau := \tau$$

$$\_Env\_LRE\_x := x \quad (1)$$

```
> read "/Users/hoeij/Downloads/Conic.txt"
```

OEIS A295371:

```
> L3 := (2*x+3)*(x+4)^2*tau^3-(2*x+3)*(7*x^2+52*x+97)*tau^2-3*(2*x+7)*(7*x^2+18*x+12)*tau+27*(2*x+7)*(x+1)^2;
```

$$L3 := (2x+3)(x+4)^2\tau^3 - (2x+3)(7x^2+52x+97)\tau^2 - 3(2x+7)(7x^2+18x+12)\tau + 27(2x+7)(x+1)^2 \quad (2)$$

```
> G, Ginv, L2, r := ReduceOrder(L3);
```

$$G, Ginv, L2, r := (x^4+9x^3+30x^2+45x+27)\tau^2 - \frac{2(4x^5+41x^4+166x^3+336x^2+345x+144)\tau}{2x+3} - \frac{9(10x^5+79x^4+251x^3+404x^2+327x+105)}{2x+3}, -\frac{(x+3)(2x+3)(x+2)\tau}{216(4x^2+28x+45)(x^2+2x+1)} - \frac{10x^3+59x^2+109x+66}{216(4x^2+20x+21)(x^2+2x+1)}, \tau^2 + \tau - \frac{3(x+2)}{4x}, \frac{4(x^2+5x+7)(2x+7)x^2}{(x^2+3x+3)(2x+5)(x+1)(x+2)} \quad (3)$$

The paper:
Yongjae Cha, Mark Van Hoeij, and Giles Levy. 2010. Solving recurrence relations using local invariants, ISSAC'2010.
gives an algorithm that tries to solve L2, or reduce it to an OEIS entry. It reduces L2 to OEIS entry A002426 which has the following recurrence:

```
> La := LREtools[RecurrenceToOperator](a(x)-((2*x-1)*a(x-1)+3*(x-1)*a(x-2))/x=0, a(x));
```

$$La := \tau^2 - \frac{(2x+3)\tau}{x+2} - \frac{3x+3}{x+2} \quad (4)$$

```
> read "/Users/hoeij/Downloads/Hom.txt" :
> Hom(symmpower(La, 2), L3);
```

$$\left[-\frac{81\tau}{10} - \frac{243}{10}\right] \quad (5)$$

```
> primpart(%[1], {tau, x});
```

$$-\tau - 3 \quad (6)$$

After comparing initial conditions this leads G := (tau + 3)/4 which then gives the formula in section

5.3 in our paper.