```
> restart:
> read '/M/m/findrel_v2.7':
```

Liouvillian example

Sequence A099364 from Sloane's database is "An inverse Chebyshev transform of (1-x)^2" and it satisfies:

```
> L := (-8-4*n)*u(n)+2*u(n+1)+(n+6)*u(n+2);
```

$$L := (-8-4\,n)\,u(n) + 2\,u(n+1) + (n+6)\,u(n+2)$$

```
> rsolve(L, u(n));
```

$$rsolve((-8-4\,n)\,u(n) + 2\,u(n+1) + (n+6)\,u(n+2), u(n))$$

```
> findrel(L, u(n));
```

$$-\frac{4\,(n+2)\,\_v(n)}{7+n} + \_v(n+2) = 0,\ u(n) = \left(\frac{1}{6}\,n + \frac{5}{6}\right)\_v(n) - \frac{1}{12}\,\_v(n+1)\,(n+6)$$

```
> Liouv := %[1];
```

$$Liouv := -\frac{4\,(n+2)\,\_v(n)}{7+n} + \_v(n+2) = 0$$

```
> rsolve(Liouv, _v(n));
```

$$\begin{cases} \dfrac{15}{8}\,\dfrac{4^{\frac{1}{2}n}\,\sqrt{\pi}\,\Gamma\left(\frac{1}{2}\,n+1\right)\_v(0)}{\Gamma\left(\frac{1}{2}\,n+\frac{7}{2}\right)} & n::even \\[2ex] \dfrac{12\,4^{\frac{1}{2}n-\frac{1}{2}}\,\Gamma\left(\frac{1}{2}\,n+1\right)\_v(1)}{\sqrt{\pi}\,\Gamma\left(\frac{1}{2}\,n+\frac{7}{2}\right)} & n::odd \end{cases}$$

```
>
```

2F1 example

Sequence A005572 from Sloane's database represents the "Number of walks on cubic lattice starting and finishing on the xy plane and never going below it" and it satisfies

```
> L := (12*n+12)*u(n)+(-20-8*n)*u(n+1)+(n+4)*u(n+2); #1, 4, 17,
  76, 354, 1704, 8421 w/offset 0
```

$$L := (12\,n+12)\,u(n) + (-20-8\,n)\,u(n+1) + (n+4)\,u(n+2)$$

```
> f := findrel(L, u(n), {u(0)=1, u(1)=4});
```

$$f := u(n) = \frac{2}{3}\,\frac{1}{n+2}\left(\sqrt{3}\,2^n\left(2\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, n+2\right], [1], \frac{2}{3}\right) - 3\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, n+1\right], [1], \frac{2}{3}\right)\right)\right), 0 \le n$$

```
> seq(simplify(eval(f[1], n=j)), j=0..4);
```

$$u(0) = 1, u(1) = 4, u(2) = 17, u(3) = 76, u(4) = 354$$

```
>
```

Legendre example

Sequence A108095 from Sloane's database represents "Coefficients of series whose square is the weight enumerator of the [8,4,4] Hamming code" and it satisfies (n-1)u(n) + (7+14n)u(n+1) + (n+2) u(n+2) = 0

```
> Lseq:=[1, 7, -24, 168, -1464, 14280, -149208, 1633128,
  -18483576, 214552968, -2540241816]: # offset = 0
> f := findrel((n-1)*u(n)+(7+14*n)*u(n+1)+(n+2)*u(n+2), u(n),
  {u(0)=1, u(1)=7, u(2)=-24, u(3)=168});
```

$$f := u(n) = \frac{(7+7\,n)\,\mathrm{LegendreP}(n+1,-7) + (97\,n+49)\,\mathrm{LegendreP}(n,-7)}{(n-1)\,n}, 2 \le n$$

```
> seq(simplify(eval(f[1], n=j)), j=2..6);
```

$$u(2) = -24, u(3) = 168, u(4) = -1464, u(5) = 14280, u(6) = -149208$$

Legendre functions of the first kind is but one of many additional special functions that we solve. Among the others currently included are:
Bessel (first and second kind), Whittaker (W and M), and types of Jacobi, Legendre, Laguerre, and Gegenbauer functions.

```
>
```

If we want a result in terms of the database:

```
> f := findrel((n-1)*u(n)+(7+14*n)*u(n+1)+(n+2)*u(n+2), u(n),
  {u(0)=1, u(1)=7, u(2)=-24, u(3)=168,[0,0,0,1]});
```

$$f := u(n) = 7\,(-1)^n \left(\frac{1}{7}\,\frac{(97\,n+49)\,A084768(n)}{n\,(n-1)} - \frac{(n+1)\,A084768(n+1)}{n\,(n-1)} \right), 3 \le n$$

```
>
```

Database example

Sequence A122031 from Sloane's database is the number of fixed points in all involutions (= self-inverse permutations) of {1,2,...,n+1}. This sequence satisfies a(n+2) = ((n+3)/(n+2))*a(n+1) + (n+3)*a(n).

```
> Lseq:=[1, 2, 6, 16, 50, 156, 532]: # offset = 0
> f := findrel(a(n+2) = ((n+3)/(n+2))*a(n+1) + (n+3)*a(n), a(n)
  , {seq(a(k-1) = Lseq[k], k=1..3)});
```

$$f := a(n) = (n+1)\,A000085(n), 1 \le n$$

```
> seq(simplify(eval(f[1], n=j)), j=0..4);
```

$$a(0) = 1, a(1) = 2, a(2) = 6, a(3) = 16, a(4) = 50$$

```
> seq(A000085(i), i=0..10);
```

1, 1, 2, 4, 10, 26, 76, 232, 764, 2620, 9496

Sequence A000085 from Sloane's database (OEIS) represents the "Number of self-inverse permutations on n letters, also known as involutions; number of Young tableaux with n cells." A00085 has much more information including many references and formulas.

```
> 'A000085(n+2)' = A000085(n+2);
  'A000085(10)' = A000085(10);
```

$$A000085(n+2) = (n+1)\,A000085(n) + A000085(n+1)$$

$$A000085(10) = 9496$$

The world wide web URL at the OEIS for A000085 is: http://www.research.att.com/~njas/sequences/A000085

NOTE about the Database: If initial conditions are not input then we get an output like so:

```
> findrel(a(n+2) = ((n+3)/(n+2))*a(n+1) + (n+3)*a(n), a(n));
```

$$a(n) = \_c_1\,(n+1)\,A000085(n) + \_c_2\,(-A000932(n) + A000932(n+1))$$

Here a(n) is returned as a linear combination of two linearly independent sequences; if we did not find more than one linearly independent sequence in Sloane's database then:

```
> findrel((-8-8*n)*a(n)+(-6-4*n)*a(n+1)+(n+2)*a(n+2), a(n));
Warning, not all solutions found
```

$$a(n) = \_c\,A084609(n)$$

Here we only have one linearly independent representative from the grouping. The warning lets us know that we have only found one solution, but we still express the relationship between the solutions of (-8-8*n)*a(n)+(-6-4*n)*a(n+1)+(n+2)*a(n+2) and the solutions of the equation satisfied by A084609.

```
>
```

Above we obtained the following output in terms of the Legendre function of the first kind :

```
> f := findrel((n-1)*u(n)+(7+14*n)*u(n+1)+(n+2)*u(n+2), u(n), {u(0)=1, u(1)=7, u(2)=-24, u(3)=168});
```

$$f := u(n) = \frac{(97\,n+49)\,\mathrm{LegendreP}(n, -7) + (7+7\,n)\,\mathrm{LegendreP}(n+1, -7)}{(n-1)\,n}, 2 \le n$$

When we want different forms we can, of course, use convert procedures in the CAS:

```
> convert(rhs(f[1]), hypergeom);
```

$$\frac{(97\,n+49)\,\mathrm{hypergeom}([-n, n+1], [1], 4) + (7+7\,n)\,\mathrm{hypergeom}([n+2, -n-1], [1], 4)}{(n-1)\,n}$$

We can also use our program to try to find a related sequence from the database:

```
> f := findrel((n-1)*u(n)+(7+14*n)*u(n+1)+(n+2)*u(n+2), u(n), {u(0)=1, u(1)=7, u(2)=-24, u(3)=168,[0,0,0,1,'g']});
```

$$f := u(n) = 7\,(-1)^n \left(\frac{1}{7}\,\frac{(97\,n+49)\,A084768(n)}{n\,(n-1)} - \frac{(n+1)\,A084768(n+1)}{n\,(n-1)}\right), 3 \le n$$

```
> g;
```

[[*Related (by recurrence, not necessarily by sequence) Sloane entries are given as:*, [*name, offset, guessed recurrence relation, [sequence]*]], [*A131763*, 0, $n\,u(n) + (-21-14\,n)\,u(n+1) + (3+n)\,u(n+2)$, [1, 3, 21, 183, 1785, 18651, 204141, 2310447, 26819121, 317530227, 3819724293, 46553474919, 573608632233, 7133530172619, 89423593269213, 1128765846337887, 14334721079385441, 183021615646831587]], [*A103211*, 0, $n\,u(n) + (-21-14\,n)\,u(n+1) + (3+n)\,u(n+2)$, [1, 4, 28, 244, 2380, 24868, 272188, 3080596, 35758828, 423373636, 5092965724, 62071299892, 764811509644, 9511373563492, 119231457692284, 1505021128450516, 19112961439180588, 244028820862442116]], [*A108095*, 0, $(n-1)\,u(n) + (7+14\,n)\,u(n+1) + (n+2)\,u(n+2)$, [1, 7, -24, 168, -1464, 14280, -149208, 1633128, -18483576, 214552968, -2540241816, 30557794344, -372427799352, 4588869057864, -57068241380952, 715388746153704, -9030126770703096, 114677768635083528, -1464172925174652696, 18783553808927819688, -242002474839216810168]], [*A084768*, 0, $(n+1)\,u(n) + (-21$

$-14\,n)\,u(n+1)+(n+2)\,u(n+2)$, [1, 7, 73, 847, 10321, 129367, 1651609, 21360031, 278905249, 3668760487, 48543499753, 645382441711, 8614382884849, 115367108888311, 1549456900170553, 20861640747345727, 281483386791966529]]]

More examples

```
> L := -8*n*(3*n+7)*(n+1)*u(n)+(-108-191*n-112*n^2-21*n^3)*u
  (n+1)+(3*n+4)*(n+3)^2*u(n+2);
```

$$L := -8\,n\,(3\,n+7)\,(n+1)\,u(n)+\left(-108-191\,n-112\,n^2-21\,n^3\right)u(n+1)+(3\,n+4)\,(3+n)^2\,u(n+2)$$

```
> findrel(L, u(n), {u(1)=1,u(2)=3});
```

$$u(n)=-\frac{1}{3}\,\frac{A000172(n)}{n}+\frac{1}{6}\,\frac{A000172(n+1)}{n},\ 1\le n$$

```
>
> L := 4*(2*n+3)*(4*n+9)*(1+n)*u(n)-4*(4*n+7)*(6*n^2+21*n+17)*u
  (1+n)+(5+2*n)*(5+4*n)*(n+2)*u(n+2);
```

$$L := 4\,(2\,n+3)\,(4\,n+9)\,(n+1)\,u(n)-4\,(4\,n+7)\left(6\,n^2+21\,n+17\right)u(n+1)+(2\,n+5)\,(4\,n+5)\,(n+2)\,u(n+2)$$

```
> findrel(L, u(n), {u(0)=1, u(1)=8, u(2)=74});
```

$$u(n)=\frac{4\,(2\,n+1)\,n\,A126765(n)}{(4\,n+5)\,(n+1)}+\frac{A126765(n+1)}{4\,n+5},\ 2\le n$$

```
>
> # _All_2nd_irr was read in the first time we called findrel
  with a symbol as the 5th option
> i:=6:
  offset := _All_2nd_irr[i][2]:
  input_name = _All_2nd_irr[i][1];
```

$$input\_name = A054768$$

```
> findrel(_All_2nd_irr[i][3], u(n), {seq(u(j)=_All_2nd_irr[i]
  [4][j+1-offset], j=offset..offset+10)});
```

$$u(n)=-2\left(\frac{1}{2}\right)^n\Gamma(n)\left(n\,(n-1)\,A025227(n)+\frac{1}{2}\,(n+1)\,n\,A025227(n+1)\right)+3\,A054765(n),\ 2\le n$$

```
>
> i:=10:
  offset := _All_2nd_irr[i][2]:
  input_name = _All_2nd_irr[i][1];
```

$$input\_name = A025228$$

```
> findrel(_All_2nd_irr[i][3], u(n), {seq(u(j)=_All_2nd_irr[i]
  [4][j+1-offset], j=offset..offset+10)});
  findrel(_All_2nd_irr[i][3], u(n), {seq(u(j)=_All_2nd_irr[i]
  [4][j+1-offset], j=offset..offset+10), [0,0,0]});
```

$$u(n) = \frac{1}{6} \frac{\sqrt{3}\, 2^n \left(2 \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, n\right], [1], \frac{2}{3}\right) - 3 \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, n-1\right], [1], \frac{2}{3}\right)\right)}{n}, 2 \le n$$

$$u(n) = \frac{1}{36} \frac{\left(13 n^2 + 26 n + 12\right) A005572(n)}{n (n-1)} - \frac{1}{36} \frac{\left(2 n^2 + 7 n + 3\right) A005572(n+1)}{n (n-1)}, 2 \le n$$

A005572 is the input from our 2F1 example above.

>

>