Study Homework Questions 3 Numerical Optimization Fall 2023

Problem 3.1

(Problem 1 on page 28 of Luenberger and Ye 3rd Ed.)

(3.1.a) Convert to standard form

$$\begin{aligned} \min \quad & x + 2y + 3z \\ \text{subject to:} \quad & \\ & 2 \leq x + y \leq 3 \\ & 4 \leq x + z \leq 5 \\ & x \geq 0, \quad y \geq 0, \quad z \geq 0 \end{aligned}$$

(3.1.b) Convert to standard form

$$\begin{aligned} \min \quad & x + y + z \\ \text{subject to:} \quad & x + 2y + 3z = 10 \\ & x \geq 1, \quad y \geq 2, \quad z \geq 1 \end{aligned}$$

Problem 3.2

(Problem 2 on page 28 of Luenberger and Ye 3rd Ed.)

A manufacturer wishes to produce an alloy that is, by weight, 30% metal A and 70% metal B. Five alloys are available at various prices as indicated below:

Alloy	1	2	3	4	5
% A	10	25	50	75	95
% B	90	75	50	25	5
Price$	5	4	3	2	1.50

The desired alloy will be produced by combining some of the other alloys. The manufacturer wishes to find the amounts of the various alloys needed and to determine the least expensive combination. Formulate this as a linear program.

Problem 3.3

(Problem 3 on page 28 of Luenberger and Ye 3rd Ed.)

An oil refinery has two sources of crude oil: a light crude that costs $35 per barrel and a heavy crude that costs $30 per barrel. The refinery produces gasoling, heating oil and jet fuel from crude oil in the amounts per barrel indicated in the following table:

Type	gas	heating	jet
Light	0.3	0.2	0.3
Heavy	0.3	0.4	0.2

The refinery has contracted to supply $900,000$ barrels of gasoline, $800,000$ barrels of heating oil, and $500,000$ barrels of jet fuel. The refinery wishes to find the amounts of light and heavy crude to purchase so as to be able to meet its obligation at minimum cost. Formulate this as a linear program.

Problem 3.4

(Problem 4 on page 28 of Luenberger and Ye 3rd Ed.)

A small firm specializes in making five types of spare automobile parts. Eah part is first cast from iron in the casting shop and then sent to the finishing shop where holes are drilled, surfaces are turned, and edges are ground. The required worker-hours (per 100 units) for each of the parts o fthe two shops are shown below:

Part	1	2	3	4	5
Casting	2	1	3	3	1
Finishing	3	2	2	1	1

The profits from parts are \$30, \$20, \$40, \$25, and \$10 (per 100 units), respectively. The capacities of the casting and finishing shops over the next month are 700 and 1000 worker-hours, respectively. Formualte the problem of determining the quantities of each spare part to be made during the month so as to maximize profit.

Problem 3.5

(Problem 5 on page 29 of the Luenberger and Ye 3rd Ed.)

Convert the following to standard form and solve:

$$\begin{aligned} \text{max} \quad & x_1 + 4x_2 + x_3 \\ \text{subject to} \quad & \\ & 2x_1 - 2x_2 + x_3 = 4 \\ & x_1 - x_3 = 1 \\ & x_2 \geq 0 \\ & x_3 \geq 0 \end{aligned}$$

Problem 3.6

(Problem 8 on page 30 of Luenberger and Ye 3rd Ed.)

Convert the following to standard form and solve:

$$\begin{aligned}
\min \quad & |x| + |y| + |z| \\
\text{subject to} \quad & \\
& x + y \leq 1 \\
& 2x + z = 3
\end{aligned}$$

Problem 3.7

(Problem 13 on page 30 of Luenberger and Ye 3rd Ed.)

Suppose x is a feasible solution for a linear program with matrix $A \in \mathbb{R}^{m \times n}$ with $m < n$ linearly independent rows. Show that there is a feasible solution y with the same cost function value, i.e., $c^T y = c^T x$, and having at most $m + 1$ positive components.

Problem 3.8

(Problem 14 on page 30 of Luenberger and Ye 3rd Ed.)

Put the following constraints for a linear program into standard form using slack variables and determine all basic feasible solutions.

$$\xi_1 + \frac{8}{3}\xi_2 \leq 4$$

$$\xi_1 + \xi_2 \leq 2$$

$$2\xi_1 \leq 3$$

$$\xi_1 \geq 0, \quad \xi_2 \geq 0$$

Problem 3.9

(Problem 8 on page 71 of Luenberger and Ye 3rd Ed.)

Use the simplex method to solve the linear program

$$\begin{aligned} \max \quad & c^T x \\ \text{subject to} \quad & \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$c = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

Problem 3.10

(Problem 10 on page 71 of Luenberger and Ye 3rd Ed.)

Use the simplex method to solve the linear program

$$\begin{aligned} \min \quad & 2\xi_1 + 4\xi_2 + \xi_3 + \xi_4 \\ \text{subject to} \quad & \\ & \xi_1 + 3\xi_2 + \xi_4 \leq 4 \\ & 2\xi_1 + \xi_2 \leq 3 \\ & \xi_2 + 4\xi_3 + \xi_4 \leq 3 \end{aligned}$$

Problem 3.11

(Part of Problem 12 on page 72 of Luenberger and Ye 3rd Ed.)

Put the following problem in standard form

$$\begin{aligned} \min \quad & \xi_1 - 3\xi_2 - 0.4\xi_3 \\ \text{subject to} \quad & \\ & \xi_1 \geq 0,\ \xi_2 \geq 0,\ \xi_3 \geq 0 \\ & 3\xi_1 - \xi_2 + 2\xi_3 \leq 7 \\ & -2\xi_1 + 4\xi_2 \leq 12 \\ & -4\xi_1 + 3\xi_2 + 3\xi_3 \leq 14 \end{aligned}$$

Determine an optimal solution and determine how many optimal solutions exist for the problem.

Problem 3.12

(Problem 18 on page 73 of Luenberger and Ye 3rd Ed.)

Solve the two linear programs using the two-phase simplex procedure:

$$\begin{aligned} \min \quad & -3\xi_1 + \xi_2 + 3\xi_3 - \xi_4 \\ \text{subject to} \quad & \\ & \xi_1 \geq 0,\ \xi_2 \geq 0,\ \xi_3 \geq 0,\ \xi_4 \geq 0 \\ & \xi_1 + 2\xi_2 - \xi_3 + \xi_4 = 0 \\ & 2\xi_1 - 2\xi_2 + 3\xi_3 + 3\xi_4 = 0 \\ & \xi_1 - \xi_2 + 2\xi_3 - 1\xi_4 = 0 \end{aligned}$$

and

$$\begin{aligned} \min \quad & \xi_1 + 6\xi_2 - 7\xi_3 + \xi_4 + 5\xi_5 \\ \text{subject to} \quad & \\ & \xi_1 \geq 0,\ \xi_2 \geq 0,\ \xi_3 \geq 0,\ \xi_4 \geq 0,\ \xi_5 \geq 0 \\ & 5\xi_1 - 4\xi_2 + 13\xi_3 - 2\xi_4 + \xi_5 = 20 \\ & \xi_1 - \xi_2 + 5\xi_3 - \xi_4 + \xi_5 = 0 \end{aligned}$$