# Graded Homework 4 Numerical Optimization Fall 2023

**The solutions are due by 11:59PM on December 11, 2023**

# Programming Exercise

## 4.1.a The Codes

(i) Implement the Gradient Projection Algorithm for two types of constraints:

- bound constraints in $\mathbb{R}^n$
- $\mathcal{K} = \{x \in \mathbb{R}^n \mid \|x\|^2 \leq 1\}$

(ii) Implement the Projected Newton Algorithm for bound constraints in $\mathbb{R}^n$.

## 4.1.a Comments on the Codes

(i) Your Gradient Projection Algorithm code should use one or both of two step selection mechanisms:

- Assuming the gradient is Lipschitz continuous with constant $L$, use a fixed step size in the interval $(0, 2/L)$. Note that for quadratic problems with matrix $A$, $L$ can be taken as $2\|A\|$.
- Backtracking with the norm form of the Armijo condition with various initial stepsizes.

(ii) You may use either the projection arc form or the feasible direction form (for which you should discuss your results for different values of $\sigma$).

(iii) Your Projected Newton Algorithm code should use backtracking with the norm form of the Armijo condition.

## 4.1.a Comments on the Codes

(i) Create various cost functions for which you know the minimizers, e.g., you can include strictly convex cost functions with global minimizers inside the constraint set, outside the constraint set and on the boundary of the constraint set. List these in your solutions and justify that the points you claim are in fact minimizers.

(ii) You may concentrate on problems in $\mathbb{R}^2$ and $\mathbb{R}^3$ in your experiments.

(iii) Run each of your problems with many different initial vectors $x_0$ and describe the behavior of each algorithm on each problem. Discuss what you observe.

(iv) Your experiments can run without a termination criterion in order to observe the behavior of $f(x)$, $\|\nabla f(x)\|$, the Gradient mapping of Beck $\|G_\mu(x)\|$, $\|x_{k+1} - x_k\|$, and $\|x_k - x(1)\|$. However, attempt to terminate with the appropriate conditions for some experiments.