1. The IVP always has a solution if f is continuous in a small rectangle containing x_0. True: f being continuous is enough to conlcude that there's **a** solution; more is needed for the solution to be unique.

2. The IVP always has a *unique* solution if f is continuous in a small rectangle containing x_0. False: f being continuous is enough to conlcude that there's **a** solution; more is needed for the solution to be unique.

3. The IVP always has a *unique* solution if $\partial f/\partial y$ is continuous in a small rectangle containing x_0. False: If f were also continuous, this would be true; however, we can't conclude anything about the continuity of f based ont he continuity of $\partial f/\partial y$. For example: Imagine that f is a function with **no** y**'s** which is *also* discontinuous: Then $\partial f/\partial y = 0$ is continuous everywhere but f is discontinuous.

4. The IVP always has a solution if f and $\partial f/\partial x$ are both continuous in a small rectangle containing x_0. True: f being continuous is enough to conlcude that there's **a** solution; more is needed for the solution to be unique.

5. The IVP always has a *unique* solution if f and $\partial f/\partial x$ are both continuous in a small rectangle containing x_0. False: This looks a lot like the existence and uniqueness theorem, except **that** theorem involves $\partial f/\partial y$ being continuous, not $\partial f/\partial x$. A counterexample to this was done in class: We wrote down three solutions to the ODE $y' = y^{1/3}$ in class despite $f = y^{1/3}$ and $\partial f/\partial x = 0$ are both continuous everhwhere.

6. The IVP may have multiple solutions. True: The autonomous ODEs we studied in §2.5 had multiple solutions (e.g. multiple equilibrium solutions).

7. The IVP may have no solution. True: This could happen in lots of situations, but one easily-imagined one is that the initial value given can't be plugged in to the solution. For example: If I had $y' = 1/x, x > 0$ as an ODE, then we can separate and integrate to get $y = \ln(x) + C$ as a general solution. If we turn this into an IVP with the initial condition $y(-5) = 2$, however, we'd have something that doesn't exist since we can't plug $x = -5$ into $y = \ln(x) + C$.

8. If the IVP has a unique solution, the existence and uniqueness theorem tells you that the solution is valid on an x-interval containing x_0. True: This is the only thing that theorem tells you about the solution...

9. If the IVP has a unique solution, the existence and uniqueness theorem helps you find the x-interval containing x_0 on which the solution is valid. False: ...and this is one of the many things that theorem **doesn't** tell you! Remember: The existence and uniqueness theorem tells you that if f and $\partial f/\partial y$ are both contiuous in a rectangle containing (x_0, y_0), then the IVP has a unique solution defined on an x-interval containing x_0; it **doesn't** tell you *which* x-interval!

10. If $f(x, y) = 0$, then the IVP has a unique solution. True: Here, $f = 0$ and $\partial f/\partial y = 0$ are both continuous everywhere, so you can use the existence and uniqueness theorem. Alternatively, you can also solve this IVP: The general solution would be $y = C$, and using the initial value $y(x_0) = y_0$ tells you that $C = y_0$, and hence that the (unique) particular solution is the constant function $y = y_0$.