

**Directions:** Answer each of the following **two (2)** questions, making sure to read the instructions for each question as you proceed.

You may use the backs of the pages for scratch work **or** get scrap paper from me!

**The last page is a bonus problem! Make sure you don't overlook it!**

1. (20 pts) Solve the IVP

$$y'' + 2y' + y = 3e^{-x}, \quad y(0) = 1, \quad y'(0) = 1.$$

**Hint:**  $(x^2 e^{kx})' = kx^2 e^{kx} + 2xe^{kx}$  and  $(x^2 e^{kx})'' = k^2 x^2 e^{kx} + 4kx e^{kx} + 2e^{kx}$  for all constants  $k$ .

SOLUTION:

- 2.(a) (6 pts) **True or False:**  $y_1(t) = e^{-2t}$  and  $y_2 = e^{3t}$  are both solutions to the second-order homogeneous ODE

$$y'' - y' - 6y = 0.$$

Justify your claim!

SOLUTION:

- (b) (4 pts) **True or False:**  $y_1(t) = e^{-2t}$  and  $y_2 = e^{3t}$  form a fundamental system of solutions for the second-order homogeneous ODE

$$y'' - y' - 6y = 0.$$

Justify your claim!

SOLUTION:

(c) (10 pts) Without using undetermined coefficients, use parts (a) and (b) to find a particular solution of the non-homogeneous ODE

$$y'' - y' - 6y = e^{-3t}.$$

Simplify fully.

SOLUTION:

**Bonus:** (5 pts) Use the method of undetermined coefficients to check the answer you got for problem 2(c).

In other words: Use the method of undetermined coefficients to find a particular solution of the non-homogeneous ODE

$$y'' - y' - 6y = e^{-3t}.$$

**Hint:** When using variation of parameters, some of the terms produced in the formula for  $Y(t)$  may have the form  $(\text{const})y_1$  or  $(\text{const})y_2$ ; to get that answer to match the one obtained using undetermined coefficients, you want to ignore any such terms! (make sure you know why!)

SOLUTION: