Quiz 2/test prep 1
(front and back)

Name: ~~ KEY ~~
(please print neatly!)

Directions: Answer each of the following **four** (4) questions, making sure to read the instructions for each question as you proceed.
You may use the backs of the pages for scratch work **or** get scrap paper from me!

1. (*10 pts*) Solve the IVP

$$\underbrace{\sin y + (x\cos y + 3y^2)y' = -2x}, \quad y(0) = \pi.$$

SOLUTION:

$$\Updownarrow$$

$$\underbrace{2x + \sin y}_{M} + \underbrace{(x\cos y + 3y^2)}_{N}y' = 0$$

$M_y = \cos y \quad N_x = \cos y \Rightarrow$ exact!

Know! There is some $f(x,y)$ such that

① $f_x = M \Rightarrow f_x = 2x + \sin y \Rightarrow f = \int \ldots dx = x^2 + x\sin y + h(y)$

② $f_y = N$

③ $f = x^2 + x\sin y + h(y)$.

- Using ③: $f_y = x\cos y + h'(y)$.
- Compare w/ ②: $f_y = x\cos y + h'(y) = x\cos y + 3y^2$

$\Rightarrow \quad h'(y) = 3y^2$

$\Rightarrow \quad h(y) = y^3$ (← IGNORE CONSTANT HERE!)

- Plug into ③: $f = x^2 + x\sin y + y^3$.

Gen Solution

$f = \text{const}$

$\Rightarrow x^2 + x\sin y + y^3 = \text{const}$

IVP:

$y(0) = \pi \Rightarrow 0^2 + 0\sin(\pi) + \pi^3 = \text{const}$

$\Rightarrow \text{const} = \pi^3$

$\Rightarrow x^2 + x\sin y + y^3 = \pi^3$

2. (*10 pts*) Find a second-order linear homogeneous differential equation whose general solution is

$$y = c_1 e^{2t} + c_2 e^{-3t}.$$

SOLUTION:

$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$ for

$r_1 = 2 \quad r_2 = -3$

$\Rightarrow$ The equation

$(r-2)(r+3)$ has these roots

$\Rightarrow$ $r^2 + r - 6$ has these roots

$\downarrow$

This is the char. eq. of

$y'' + y' - 6y = 0$

3. (*10 pts*) For which of the following initial conditions does the IVP

$$(\ln(y) - 1)\frac{dy}{dx} - 2\sin x = \ln(\ln(x)), \quad y(x_0) = y_0$$

have a unique solution? **There may be more than one!**

i. $y(1) = 4$ ii. $y\left(\frac{\pi}{4}\right) = 0$ iii. $y\left(\frac{\pi}{2}\right) = 0$ iv. $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ v. $y(e) = e$ vi. None of These

ODE $\Rightarrow$ $\frac{dy}{dx} = \boxed{\frac{\ln(\ln x) + 2\sin x}{\ln(y) - 1}}$ $f(x,y)$.

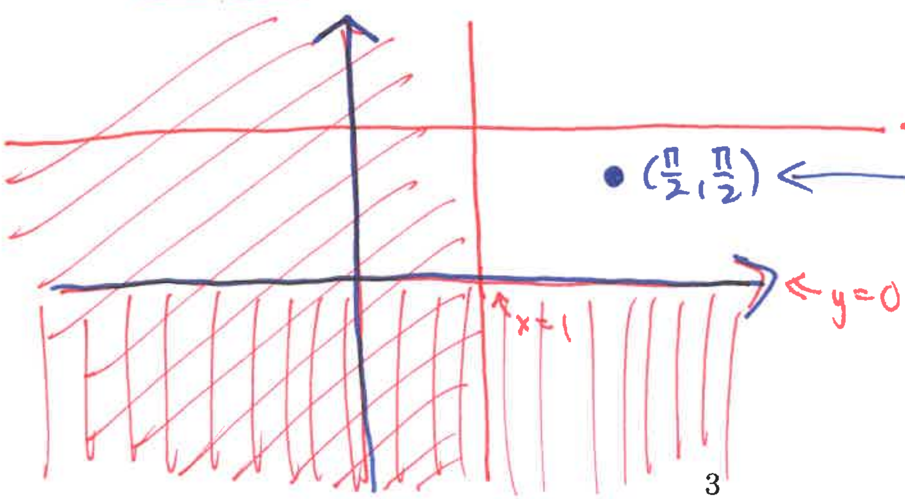
using existence & uniqueness!

- f not continuous:
  - $x \leq 0$
  - $\ln(x) \leq 0 \Rightarrow x \leq 1$
  - $y \leq 0$
  - $\ln(y) - 1 = 0 \Rightarrow \ln(y) = 1 \Rightarrow y = e$.

- $\frac{\partial f}{\partial y} = \frac{0 - [\ln(\ln x) + 2\sin x][\frac{1}{y}]}{(\ln(y) - 1)^2}$

$\hookrightarrow$ $\frac{\partial f}{\partial y}$ not continuous:
  - Same as above
  - $y = 0$

Graph of Badness (all shaded regions + lines $y=0$, $x=1$, $y=e$ are bad)



(i) Bad $(x=1)$
(ii) Bad $(y=0)$
(iii) Bad $(y=0)$
(iv) Good!
(v) Bad $(y=e)$
(vi) Bad (b/c (iv))

4. (*1 pt ea.*) Consider the first-order IVP

6 fart heads

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

Indicate whether each of the following questions is True or False by writing the words "True" or "False" (and **not** just the letters "T" or "F") **No justification is required!**

9 (a) The IVP always has a solution if f is continuous in a small rectangle containing x_0.

True

8 (b) The IVP always has a *unique* solution if f is continuous in a small rectangle containing x_0.

False

24 (c) The IVP always has a *unique* solution if $\frac{\partial f}{\partial y}$ is continuous in a small rectangle containing x_0.

False

11 (d) The IVP always has a solution if f and $\frac{\partial f}{\partial x}$ are both continuous in a small rectangle containing x_0.

True

38 (e) The IVP always has a *unique* solution if f and $\frac{\partial f}{\partial x}$ are both continuous in a small rectangle containing x_0.

False

8 (f) The IVP may have multiple solutions.

True

14 (g) The IVP may have no solution.

True

16 (h) If the IVP has a unique solution, the existence and uniqueness theorem tells you that the solution is valid on an x-interval containing x_0.

True

10 (i) If the IVP has a unique solution, the existence and uniqueness theorem helps you find the x-interval containing x_0 on which the solution is valid.

False

28 (j) If $f(x, y) = 0$, then the IVP has a unique solution. $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow y = \text{const}$ & $y(x_0) = y_0$

True

$\Rightarrow y = y_0$ is the unique sol'n!