

Quiz 2/test prep 1
(front and back)

Name: _____
(please print neatly!)

Directions: Answer each of the following four (4) questions, making sure to read the instructions for each question as you proceed.

You may use the backs of the pages for scratch work **or** get scrap paper from me!

1. (10 pts) Solve the IVP

$$\sin y + (x \cos y + 3y^2)y' = -2x, \quad y(0) = \pi.$$

SOLUTION:

2. (10 pts) Find a second-order linear homogeneous differential equation whose general solution is

$$y = c_1 e^{2t} + c_2 e^{-3t}.$$

SOLUTION:

3. (10 pts) For which of the following initial conditions does the IVP

$$(\ln(y) - 1) \frac{dy}{dx} - 2 \sin x = \ln(\ln(x)), \quad y(x_0) = y_0$$

have a unique solution? **There may be more than one!**

- i. $y(1) = 4$ ii. $y\left(\frac{\pi}{4}\right) = 0$ iii. $y\left(\frac{\pi}{2}\right) = 0$ iv. $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ v. $y(e) = e$ vi. None of These

4. (1 pt ea.) Consider the first-order IVP

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

Indicate whether each of the following questions is True or False by writing the words “True” or “False” (and **not** just the letters “T” or “F”). **No justification is required!**

- (a) The IVP always has a solution if f is continuous in a small rectangle containing x_0 .
- (b) The IVP always has a *unique* solution if f is continuous in a small rectangle containing x_0 .
- (c) The IVP always has a *unique* solution if $\frac{\partial f}{\partial y}$ is continuous in a small rectangle containing x_0 .
- (d) The IVP always has a solution if f and $\frac{\partial f}{\partial x}$ are both continuous in a small rectangle containing x_0 .
- (e) The IVP always has a *unique* solution if f and $\frac{\partial f}{\partial x}$ are both continuous in a small rectangle containing x_0 .
- (f) The IVP may have multiple solutions.
- (g) The IVP may have no solution.
- (h) If the IVP has a unique solution, the existence and uniqueness theorem tells you that the solution is valid on an x -interval containing x_0 .
- (i) If the IVP has a unique solution, the existence and uniqueness theorem helps you find the x -interval containing x_0 on which the solution is valid.
- (j) If $f(x, y) = 0$, then the IVP has a unique solution.