**Quiz 5**/test prep 1
(front and back)

Name: ~KEY~
(please print neatly!)

**Directions:** Answer each of the following questions. Make sure to read the instructions for each question as you proceed. *For multiple choice questions, indicate your choice(s) by circling/drawing a box around the appropriate selection(s).*

1. Which of the following is a general solution of the differential equation $(1+x^2)dy = \dfrac{dx}{3y^2-1}$? $(3y^2-1)dy = \frac{dx}{1+x^2}$

(a) $y^3 + y - \arctan x = C$

(b) $y^3 - y - C\arctan x = 0$

(c) $y^3 - y - \tan x = C$

(d) $y^3 - y - \arctan x = C$

(e) $y = (\arctan x + C)^{1/3}$

(f) $y^3 - y = \arctan(\arctan(Cx))$

2. Let $r$, $T$, $K$, and $\ell$ be constants for which $r > 0$ and $0 < K < \ell < T$. Select **all** of the following values $y$ which are equilibrium solutions of the autonomous ODE

$$\frac{dy}{dx} = -r\left(1-\frac{y}{\ell}\right)\left(2+\frac{y}{KT}\right)(y^3 - y^2 - 2y).$$

$y = \ell$, $y = -2KT$, $y = 0$, $y = 2$, $y = -1$

$y(y^2 - y - 2)$

(a) $y = 0$

(b) $y = 1$

(c) $y = -1$

(d) $y = -2$

(e) $y = -2KT$

(f) $y = \ell$

(g) $y = KT$

(h) $y = 2KT$

3. $m(x) = \dfrac{2}{x^3}$ is an integrating factor for which of the following ($\geq 1$) linear ODEs?

**Hint**: If $k(x)$ is an integrating factor of a linear ODE, then so is $c \cdot k(x)$ for all constants $c$.

$m = e^{\int \frac{3}{x}dx} = x^3$

(a) $xy' + 3y = 2x^3$

(b) $x^2y' - 3x^3y = 2x^3$

$n = e^{\int -3x\,dx} = e^{-\frac{3}{2}x^2}$

$n = e^{\int -\frac{3}{x}} = \frac{1}{x^3}$

(c) $xy' - 3y = 2x^3$

(d) $xy' + 3y = 0$ $\rightsquigarrow m = e^{\int \frac{3}{x}dx} = x^3$

(e) $-x^2y' + 3xy = 2x^3$ $\rightsquigarrow m = e^{\int -\frac{3}{x}dx} = \frac{1}{x^3}$

(f) $xy' - 3y = 0$ $\rightsquigarrow m = e^{\int -\frac{3}{x}dx} = \frac{1}{x^3}$

4. Consider the IVP

$$(x(x-1))\frac{dy}{dx} + \ln(x+5)y = \sqrt{2-\frac{3}{x}}, \quad y(\pi) = -4.$$

$x_0$

On what interval is the solution to this problem valid? **Do not attempt to solve!**

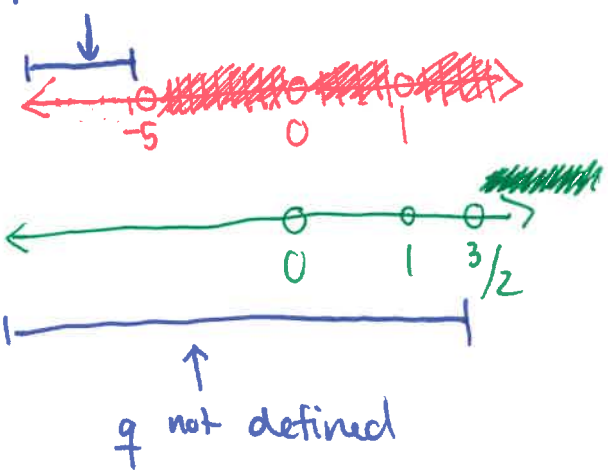
Ans: $(3/2, \infty)$

$$\frac{dy}{dx} + \frac{\ln(x+5)}{x(x-1)}y = \frac{\sqrt{2-3/x}}{x(x-1)}$$

- $x \neq 0$
- $x \neq 1$
- $x+5 > 0$ $\rightsquigarrow x > -5$

- $x \neq 0$
- $x \neq 1$
- $2 - \frac{3}{x} \geq 0$ $\rightsquigarrow 2 \geq \frac{3}{x}$ $\rightsquigarrow x \geq \frac{3}{2}$

p not defined




q not defined

The region shaded red and green is $(3/2, \infty)$, and since $x_0 = \pi$ in $(3/2, \infty)$, the ODE/IVP will have a solution there!

Scratch Paper