

Directions: Answer each of the following questions. Make sure to read the instructions for each question as you proceed. *For multiple choice questions, indicate your choice(s) by circling/drawing a box around the appropriate selection(s).*

1. Which of the following is a general solution of the differential equation $(1 + x^2)dy = \frac{dx}{3y^2 - 1}$?

(a) $y^3 + y - \arctan x = C$

(d) $y^3 - y - \arctan x = C$

(b) $y^3 - y - C \arctan x = 0$

(e) $y = (\arctan x + C)^{1/3}$

(c) $y^3 - y - \tan x = C$

(f) $y^3 - y = \arctan(\arctan(Cx))$

2. Let r , T , K , and ℓ be constants for which $r > 0$ and $0 < K < \ell < T$. Select **all** of the following values y which are equilibrium solutions of the autonomous ODE

$$\frac{dy}{dx} = -r \left(1 - \frac{y}{\ell}\right) \left(2 + \frac{y}{KT}\right) (y^3 - y^2 - 2y).$$

(a) $y = 0$

(e) $y = -2KT$

(b) $y = 1$

(f) $y = \ell$

(c) $y = -1$

(g) $y = KT$

(d) $y = -2$

(h) $y = 2KT$

3. $m(x) = \frac{2}{x^3}$ is an integrating factor for which of the following (≥ 1) linear ODEs?

Hint: If $k(x)$ is an integrating factor of a linear ODE, then so is $c \cdot k(x)$ for all constants c .

(a) $xy' + 3y = 2x^3$

(d) $xy' + 3y = 0$

(b) $x^2y' - 3x^3y = 2x^3$

(e) $-x^2y' + 3xy = 2x^3$

(c) $xy' - 3y = 2x^3$

(f) $xy' - 3y = 0$

4. Consider the IVP

$$(x(x-1))\frac{dy}{dx} + \ln(x+5)y = \sqrt{2 - \frac{3}{x}}, \quad y(\pi) = -4.$$

On what interval is the solution to this problem valid? **Do not attempt to solve!**

Scratch Paper