

• Solve the following IVP:

$$y'' - 2y' + y = xe^x + 4, \quad y(0) = 1, \quad y'(0) = 1$$

Steps:

① Find gen sol'n

(a) Find  $c_1 y_1 + c_2 y_2$  for homogeneous

(b) Find  $Y(x)$  using undetermined coeff.

(c) Add (a) + (b)

② Find  $c_1$  &  $c_2$  using initial vals

① (a)  $y'' - 2y' + y = 0 \Rightarrow r^2 - 2r + 1 = 0 \Rightarrow (r-1)(r-1) = 0 \Rightarrow r=1 \text{ \& } r=1$

$\hookrightarrow c_1 e^x + c_2 x e^x$

(b)  $g(x) = x e^x + 4$

$\hookrightarrow$  Consider:  $y'' - 2y' + y = 4 \rightarrow$  guess:  $Y = A \Rightarrow Y' = 0 \Rightarrow Y'' = 0$

$\Rightarrow 0 - 2(0) + A = 4 \Rightarrow A = 4$

$\leadsto Y_1(x) = 4.$

• Consider:  $y'' - 2y' + y = x e^x \rightarrow$  guess 1:  $(Ax+B)e^x$  *Can't use it!*

guess 2:  $x^2(Ax+B)e^x = (Ax^3+Bx^2)e^x$

$Y' = (3Ax^2+2Bx)e^x + (Ax^3+Bx^2)e^x \Rightarrow Y'' = (6Ax+2B)e^x + (3Ax^2+2Bx)e^x + (Ax^3+Bx^2)e^x$

$= (6Ax+2B)e^x + (3Ax^2+2Bx)e^x + (Ax^3+Bx^2)e^x$

plug in  
 $\Rightarrow (Ax^3+Bx^2)e^x + (6Ax+2B)e^x + (3Ax^2+2Bx)e^x + (Ax^3+Bx^2)e^x$   
 $- (2Ax^3+2Bx^2)e^x - (6Ax^2+4B)e^x$   
 $+ (Ax^3+Bx^2)e^x$

$= x e^x \Rightarrow 6Ax e^x + 2B e^x = x e^x$

$\Rightarrow B = 0 \text{ \& } A = \frac{1}{6}$

$\leadsto Y_2 = \frac{1}{6} x^3 e^x$

(c)  
 $\Rightarrow Y(x) = \frac{1}{6} x^3 e^x + 4$

Gen Sol'n:  $c_1 e^x + c_2 x e^x + \frac{1}{6} x^3 e^x + 4 = y$



② use initial vals

$$y = c_1 e^x + c_2 x e^x + \frac{1}{6} x^3 e^x + 4$$

$$\text{(i)} \quad y(0) = 1$$

$$\text{(ii)} \quad y'(0) = 1$$

$$\Rightarrow \text{(i)} \quad 1 = c_1 + 0 + 4 \Rightarrow$$

$$c_1 = -3$$

$$\text{(ii)} \quad y' = c_1 e^x + c_2 x e^x + c_2 e^x + \frac{1}{2} x^2 e^x + \frac{1}{6} x^3 e^x + 0$$

$$\Rightarrow 1 = c_1 + 0 + c_2 + 0 + 0 + 0$$

$$\Rightarrow 1 = c_1 + c_2$$

$$\Rightarrow 1 = -3 + c_2 \Rightarrow$$

$$c_2 = 4$$

Soln:

$$y = -3e^x + 4xe^x + \frac{1}{6}x^3e^x + 4.$$