Find the general solutions of the ODE:

1. $y'' + 2y' - 15y = 0$

2. $y'' + 2y' + y = 0$

2. $y'' + 2y' + 10y = 0$

§3.2 – Solutions of Linear Homogeneous Eq's & the Wronskian

<u>Recall:</u> If y_1 & y_2 are solutions to the ODE $ay''+by'+cy=0$,

then so is $y=c_1y_1+c_2y_2$.

↳ Are <u>all</u> solutions of this form?

- Suppose $y=c_1y_1+c_2y_2$ is a solution for an IVP

$$ay''+by'+cy=0 \quad y(x_0)=y_0 \quad y'(x_0)=y_0'$$

Then:

$$\left.\begin{aligned} &\circ\ y_0=c_1y_1(x_0)+c_2y_2(x_0) \\ &\circ\ y_0'=c_1y_1'(x_0)+c_2y_2'(x_0) \end{aligned}\right] \ (\star)$$

⇒ Can find c_1 & c_2 using matrices:

$$\begin{bmatrix} y_0 \\ y_0' \end{bmatrix} = \overbrace{\begin{bmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{bmatrix}}^{A} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \leftarrow \text{matrix form of } (\star)$$

has a unique solution (for c_1, c_2) iff

$$\det(A) \neq 0$$

$$\Leftrightarrow \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} \neq 0$$

Def: The Wronskian W.

$$\Leftrightarrow y_1(x_0)y_2'(x_0)-y_2(x_0)y_1'(x_0) \neq 0.$$

Thm: Let y_1, y_2 be two solns for ODE $ay''+by'+cy=0$. There exist constants c_1 & c_2 s.t.

$$y = c_1 y_1 + c_2 y_2$$

is a soln of the IVP $ay''+by'+cy=0$, $y(x_0)=y_0$, $y'(x_0)=y_0'$ iff $W(x_0) \neq 0$.

Ex: $y''+5y'+6y=0 \iff r^2+5r+6=0$

$$\iff (r+2)(r+3)=0$$

$$\iff r=-2 \quad r=-3.$$

This gives $y_1 = e^{-2x}$ & $y_2 = e^{-3x}$. Now:

$$W(x) = \begin{vmatrix} e^{-2x} & e^{-3x} \\ -2e^{-2x} & -3e^{-3x} \end{vmatrix} = -3e^{-5x} + 2e^{-5x}$$

$$= -e^{-5x}$$

So $W(x) \neq 0$ for <u>all</u> x. This mean's for <u>any</u> x_0 there exist const. c_1, c_2 s.t. the IVP

$$y''+5y'+6y=0, \quad y(x_0)=y_0, \quad y'(x_0)=y_0'$$

has a unique soln of the form

$$y = c_1 y_1 + c_2 y_2!$$

Wronskians also help you know that "general soln's" really are general.

<u>Thm:</u> If y_1 & y_2 are soln's to

$$ay''+by'+cy=0,$$

then $y=c_1y_1+c_2y_2$ includes <u>every</u> solution to the ODE iff there is some point x_0 where $W(x_0)\neq 0$.