Intro (before § 2.1)

- what is an ordinary differential equation (ODE)?

~~Calculus~~

↳ ★ What is a differential equation?

  - In calculus: It's an equation that has derivatives in it.

Ex: ① $\frac{dy}{dx} = x + y$

② $y'' + y = 7$

Assume $y(x)$ is only function & has only one var

③ $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = \sin(x+t)$. $u(x,t)$ func of two vars.

To make sense of this, you ~~[illegible]~~ look at dx, dt, etc., as functions whose inputs are vectors.

↳ • vectors are tangent vectors

• "undo" tangency by integrating, where "integral" is a line integral.

  - In ODE: It's an equation that has differentials in it.

Ex: ④ $2x\,dx = \frac{4}{x}\,dt \rightsquigarrow \frac{dx}{dt} = \frac{2}{x^2}$

⑤ $x^2 \frac{dx}{dt} = \sin(x)\cos(t) \rightsquigarrow \frac{x^2}{\sin x}\,dx = \cos t\,dt$

↑ This is all in the background & not toooo important to understanding of the class.

↳ ★ what is ordinary?

  - No partial derivatives (e.g. none of Ex ③).

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• What does it "mean" to "solve" an ODE?

↳ an ODE has ~~1~~ an independent variable (e.g. x) and a function which depends on it (e.g. $y = y(x)$).
(differentials related to)

Ex: $y'''' + y''' + y'' + y' = 0$

$$\left(\frac{d^4y}{dx^4} + \dots + \frac{dy}{dx} = 0 \right).$$

↳ The Solution to an ODE is a function y which satisfies the ODE.

"separable" ODES b/c you can clearly separate "x part" from "y part"

Ex: ① ODE: $\frac{dy}{dx} = 7$

Solution: $y(x) = 7x + C$

$\frac{dy}{dx} = 7 \Rightarrow dy = 7dx$

$\Rightarrow \int dy = \int 7dx$

$\Rightarrow y = 7x + C$

② ODE: $\frac{dy}{dx} = 7x$

solution: $y(x) = \frac{7}{2}x^2 + C$

$\frac{dy}{dx} = 7x \Rightarrow dy = 7x\,dx$

$\Rightarrow \int dy = \int 7x\,dx$

$\Rightarrow y = \frac{7}{2}x^2 + C$

③ ODE: $y'''' + y''' + y'' + y' = 0$

Solution? What do you guys think?

- Do all ODEs have a solution?

  ↳ Almost always yes, but if you're thinking like the above examples: Almost always no.

Ex: $\dfrac{dy}{dx} = e^{-x^2}$

$$\Rightarrow dy = e^{-x^2}dx \quad \Rightarrow \int dy = \int e^{-x^2}dx$$

$$\Rightarrow y(x) = \underbrace{\int e^{-x^2}dx}_{\text{No "closed form" integral!}} + C$$

Show slope field view

Ex: $\dfrac{dy}{dx} = y^2 - x$ ← Even though things are easily integrated independently, this thing requires some extremely difficult trickeration!

(show M- ; then VF)

- A good philosophy for this course / ODEs in general: ODEs define functions & the object of this course is to develop the methods for understanding (maybe computing, maybe not) these functions.

Recall:

- A solution to an ODE is an equation / function $y = \ldots$ whose derivatives satisfy the ODE.

- To verify whether an equation / function is a solution to an ODE, you take its derivatives & see if they make the ~~equation~~ ODE true.

Ex: ① True or False: $y = x^2$ is a solution to the ODE $x\frac{dy}{dx} = 2y$.

$$\hookrightarrow \frac{dy}{dx} = 2x \Rightarrow \begin{array}{l} LHS = x(2x) = 2x^2 \\ RHS = 2y = 2(x^2) \end{array}$$

Since LHS = RHS, $y = x^2$ is a solution!

② T or F: $y = x^2$ is a solution to $y'' + y' = 0$.

- $y' = 2x$ & $y'' = 2$

$$\Rightarrow \begin{array}{l} LHS = 2 + 2x \\ RHS = 0 \end{array}$$ Not always equal, so $y = x^2$ not a solution.