\# 3 Redux

ⓐ $y'' - 5y' + 6y = 0, \quad y(0)=0, \; y'(0)=1$

$\hookrightarrow \bullet\ \mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2-5s+6) - 5y(0) - s\,y(0) - y'(0)$

$= \mathcal{L}\{y\}(s^2-5s+6) - 0 - 0 - 1$

$= \mathcal{L}\{y\}(s^2-5s+6) - 1$

$\bullet\ \mathcal{L}\{RHS\} = 0$

$\Rightarrow \mathcal{L}\{y\} = \dfrac{1}{s^2-5s+6}$

ⓑ $y'' + 4y = 0, \quad y(0)=1, \; y'(0)=3$

$\hookrightarrow \bullet\ \mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2+4) - s\,y(0) - y'(0)$

$= \mathcal{L}\{y\}(s^2+1) - s - 3$

$\bullet\ \mathcal{L}\{RHS\} = 0$

$\Rightarrow \mathcal{L}\{y\} = \dfrac{s+3}{s^2+1}$

ⓒ $y'' - 5y' + 6y = \dfrac{t^3}{3!}, \quad y(0)=1, \; y'(0)=1$

$\hookrightarrow \bullet\ \mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2-5s+6) - 5y(0) - s\,y(0) - y'(0)$

$= \mathcal{L}\{y\}(s^2-5s+6) - 5 - s - 1$

$\bullet\ \mathcal{L}\{RHS\} = \dfrac{1}{s^4}$

$\Rightarrow \mathcal{L}\{y\} = \dfrac{6 + s + \frac{1}{s^4}}{(s^2-5s+6)} = \dfrac{s^5 + 6s^4 + 1}{s^4(s-3)(s-2)}$

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(d) $y'' + 4y' + 4y = e^t + t, \quad y(0) = -2, \quad y'(0) = -2$

$\hookrightarrow$ • $\mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2+4s+4) + 10 + 2s$

• $\mathcal{L}\{RHS\} = \dfrac{1}{s^2} + \dfrac{1}{s-1}$

$$\Rightarrow \mathcal{L}\{y\} = \frac{\frac{1}{s^2} + \frac{1}{s-1} - 10 - 2s}{(s^2+4s+4)} = \frac{-2s^4 - 8s^3 + 11s^2 + s - 1}{s^2(s+2)^2(s-1)}$$

(e) $y'' - 8y' - 9y = \cos(t) - \sin(2t), \quad y(0) = 0, \quad y'(0) = -1$

$\hookrightarrow$ • $\mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2-8s-9) + 1$

• $\mathcal{L}\{RHS\} = \dfrac{-2}{s^2+4} + \dfrac{s}{s^2+1}$

$$\Rightarrow \mathcal{L}\{y\} = \frac{\mathcal{L}\{RHS\} - 1}{s^2-8s-9} = \frac{-s^4 + s^3 - 7s^2 + 4s - 6}{(s^2+1)(s^2+4)(s+1)(s-9)}$$

(f) $y'' + 4y = \cos(t) - \sin(2t), \quad y(0) = 2, \quad y'(0) = -1$

$\hookrightarrow$ • $\mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2+4) - 2s + 1$

• $\mathcal{L}\{RHS\} = \dfrac{-2}{s^2+4} + \dfrac{s}{s^2+1}$

$$\Rightarrow \mathcal{L}\{y\} = \frac{\mathcal{L}\{RHS\} + 2s - 1}{s^2+4} = \frac{2s^5 - s^4 + 11s^3 - 7s^2 + 12s - 6}{(s^2+1)(s^2+4)^2}$$

(g) $y'' - 4y = te^{-3t}\cos t - \sin(2t)$, $y(0)=4$, $y'(0)=0$

$\hookrightarrow \bullet\ \mathcal{L}\{LHS\} = \mathcal{L}\{y\}(s^2-4) - 4s$

$\bullet\ \mathcal{L}\{RHS\} = \dfrac{-2}{s^2+4} + \left[\dfrac{s^2+6s+8}{(10+6s+s^2)^2}\right] = \dfrac{(s+3)^2-1}{((s+3)^2+1)^2},$

b/c $\mathcal{L}\{t\cos t\} = \dfrac{s^2-1}{(s^2+1)^2} = G(s)$ & $\mathcal{L}\{te^{-3t}\cos t\} = G(s+3)$

$\Rightarrow \mathcal{L}\{y\} = \dfrac{\mathcal{L}\{RHS\} + 4s}{s^2-4}$

$= \dfrac{4s^7 + 48s^6 + 240s^5 + 671s^4 + 1278s^3 + 1820s^2 + 1384s - 168}{(s+2)(s-2)(s^2+4)(s^2+6s+10)^2}$