Homework 4/test prep 1

(front and back)

Name: ______________________________

(please print neatly!)

Directions: Answer each of the following **three (3)** questions, making sure to read the instructions for each question as you proceed. If you use the Laplace table, make sure you specify which entry/entries you're referencing!

Make sure that your submission meets the criteria of the Homework Policy on the `Homework` tab of the course webpage!

Due date: Monday, July 31

1. Find the Laplace transform of each of the following functions.

   (a) $f(t) = e^{3t}\sin(4t)$

   (b) $f(t) = e^{3t}\sin(4t) + e^{3t}\cos(4t)$

   (c) $f(t) = e^{3t}\sin(4t)\cos(4t)$ **Hint**: Use a double-angle formula.

   (d) $f(t) = \begin{cases} 4.913 & \text{if } 0 \le t < 2 \\ e^{3t}\sin 4t & \text{if } 2 \le t < 5 \\ 2 - t & \text{if } 5 \le t < 14 \\ t^2 & \text{if } t \ge 14 \end{cases}$

   (e) $f(t) = te^t \sin t$. **Hint**: Let $g(t) = t\sin t$ and rewrite $f(t)$ as $f(t) = e^{ct}g(t)$ for some constant c.

2. Use partial fractions to find the inverse Laplace transform of each of the following functions $F(s)$, i.e. find the function $f(t)$ for which $\mathcal{L}\{f(t)\} = F(s)$.

(a) $F(s) = \dfrac{1}{s^4}$

(b) $F(s) = \dfrac{s}{(s+1)(s-1)}$

(c) $F(s) = \dfrac{1}{s^2(s+1)^2(s^2+1)^2}$

Hint: The denominator consists of repeated factors; brush up on how to handle those!

(d) $F(s) = \dfrac{8s^2 - 4s + 12}{s^2(s+1)(s^2+9)}$

(e) $F(s) = 1$

Hint: You don't know this function (err...“function”), but...tell me something about what would need to happen to make this true! Try guessing and checking some stuff, etc. etc. Think like a mathematician!

(f) $F(s) = \dfrac{s^2}{(s+1)(s-1)}$

Hint: Let $\delta(t)$ denote the function from (e) [which you don't know] and use long division!

3. Solve each of the following IVPs using Laplace transforms; then, check your answers using characteristic equations and/or undetermined coefficients. **Do not check using variation of parameters!**

   (a) $y'' - 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = 1$

   (b) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 3$

   (c) $y'' - 5y' + 6y = \frac{t^3}{3!}$, $y(0) = 1$, $y'(0) = 1$ **Hint**: Recall: $n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$

   (d) $y'' + 4y' + 4y = e^t + t$, $y(0) = -2$, $y'(0) = -2$

   (e) $y'' - 8y' - 9y = \cos t - \sin(2t)$, $y(0) = 0$, $y'(0) = -1$

   (f) $y'' + 4y = \cos t - \sin(2t)$, $y(0) = 2$, $y'(0) = -1$

   (g) $y'' - 4y = te^{-3t} \cos t - \sin(2t)$, $y(0) = 4$, $y'(0) = 0$ **Hint**: See 1(e) for how to handle $te^{-3t} \cos t$.