**Homework 3**/test prep 1 Name: _____

(front and back) (please print neatly!)

**Directions:** Answer each of the following **four (4)** questions, making sure to read the instructions for each question as you proceed.

**Make sure that your submission meets the criteria of the Homework Policy on the `Homework` tab of the course webpage!**

**Note:** Questions 1–3 are good quiz prep; all are good exam prep!

**Due date:** Monday, July 17

1. Solve the initial value problem

$$y'' + 4y = x^2 e^{-x} - x \sin x + 4x, \quad y(0) = 0, \quad y'(0) = 1.$$

SOLUTION:

2. Write down the general solution for each of the following non-homogeneous ODEs.

**Hint**: Do not use undetermined coefficients!

(a) $y'' + 4y' - 5y = 16e^{x/2}$

(b) $2y'' + 8y' + 8y = 2t^{-2}e^{-2t}, \quad t > 0$

(c) $y'' - 2y' + y = 3\sec(2t), \quad t < \dfrac{\pi}{6}$

(d) $y'' - 5y' + 6y = g(t)$ **Hint**: $g(t)$ is an arbitrary continuous function.

3. Show that the functions $y_1$ and $y_2$ satisfy the corresponding homogeneous equation; then, find a particular solution of the given non-homogeneous ODE. Throughout, assume $x > 0$.

$$x^2y'' + xy' + (x^2 - 0.25)y = 3x^{3/2}\sin(x); \quad y_1 = \frac{\sin x}{\sqrt{x}}, \quad y_2 = \frac{\cos x}{\sqrt{x}}$$

SOLUTION:

4. Find the Laplace transform for each of the following functions. Throughout, assume that $a$ and $b$ are real constants and that $i = \sqrt{-1}$ is the imaginary unit.

   (a) $f(t) = 1$

   (b) $f(t) = t^2$

   (c) $f(t) = \sin(bt)$ **Hint**: $\sin(bt) = \dfrac{e^{ibt} - e^{-ibt}}{2i}$

   (d) $f(t) = t^2 e^{at}$ **Hint**: Use integration by parts!

   (e) $f(t) = 5\sin(bt) - 2t^2 e^{at}$