Name: ~~ KEY ~~

MAP 2302 — Homework 1

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems **must** be done on scratch paper and not on this handout!* **Date Due:** Friday, May 26.

1. **Note**: Yes, you will get graded for this question. ☺

   (a) Navigate to our course homepage at

   `http://www.math.fsu.edu/~cstover/teaching/su17_map2302/`

   (b) Read and familiarize yourself with the three resources listed under *Supplementary Resources* on the General Info tab.

   (c) Follow the instructions for using Slack messenger.

   Note: This may require that I approve your email address, so to avoid some last minute glitch where I don't get to your approval on-time, please don't wait to do this!

   (d) Navigate to the channel **`#introductions`** in the left column under Channels; its browser URL should be something like **`https://summer2017-ode.slack.com/messages/random`**.

   (e) Introduce yourself by answering each of the following questions:

   Note: This will be visible to everyone who signs into our class's chat room, so you definitely want to keep these answers PG-13, safe for work, and non-incriminatory. ☺

   (i) What is your name?

   (ii) Where are you from (using any interpretation you'd like)?

   (iii) What is the best (interpret this however you'd like) place you've ever visited/lived? Why is it so special to you?

   (iv) How long have you been in Tallahassee?

   (v) What is your major?

   (vi) What do you like to do for fun? (besides differential equations, of course!)

   (vii) What is the coolest math/science "thing" you know? Why is it interesting to you?

   (f) Under which username did you register for Slack? ______________________

2. For each of the following differential equations, determine whether it's separable, linear, or neither. If it's separable, separate it; if it's linear, compute its integrating factor. **Do not solve!**

(a) $y'' + y' = 2y + \sin x$. $\boxed{\text{neither}}$ (b/c y'')

(b) $2xy' - x^2 y = \sin x + e^x$. $\boxed{\text{linear}}$

$\Rightarrow y' - \frac{x}{2} = \frac{\sin x + e^x}{2x}$ $\quad \hookrightarrow m(x) = \exp\left(\int \frac{-x}{2} dx\right) = \exp\left(\frac{-x^2}{4}\right)$

(c) $y' = x^3 \sin y$. $\boxed{\text{separable}}$

$\hookrightarrow \frac{dy}{\sin y} = x^3 dx$

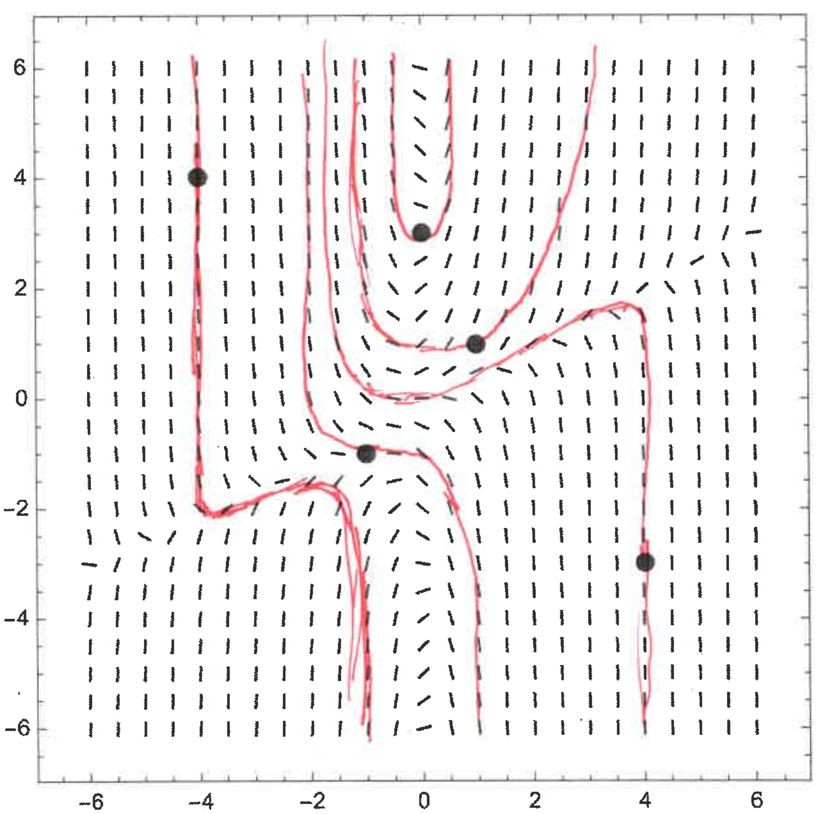
(d) $y' = x^3 \sin y - y$. $\boxed{\text{Neither}}$

(e) $y' = x^3 \sin x - y$. $\boxed{\text{Linear}}$

$\Rightarrow y' + y = x^3 \sin x$ $\quad \hookrightarrow m(\cancel{x}) = \exp\left(\int 1 dx\right) = e^x$

3. Given the slope field below, draw the (approximate) integral curves passing through each of the indicated points.

- The aim here is to connect the tangent vectors in a coherent way through each point.
- Nobody's will be perfect.
- It's a guesstimate & more art than science!
- I'll put a screen-shot of computer renderings online.



4. Each of the following questions relates to the equation $\frac{dy}{dx} = 3(1+y^2)\sec^2(x)$.

(a) Is this ODE separable, linear, or other? How do you know?

separable: $\boxed{\frac{dy}{1+y^2} = 3\sec^2 x \, dx}$

(b) Compute the general solution for this ODE.

$$\boxed{y = \tan(3\tan x + C)}$$

(c) Find the particular solution of this ODE subject to the initial condition $\underbrace{y(0) = 1}_{\Downarrow}$.

$C = \frac{\pi}{4}$

So: $\boxed{y = \tan\left(3\tan x + \frac{\pi}{4}\right)}$

(d) Determine the interval in which the solution in part (c) exists.

Hint: The part of the domain of $\tan x$ containing $x = 0$ is $(-\pi/2, \pi/2)$.

y is valid where defined:

$$-\frac{\pi}{2} < 3\tan x + \frac{\pi}{4} < \frac{\pi}{2}$$

$$\Rightarrow -\frac{3\pi}{4} < 3\tan x < \frac{\pi}{4}$$

$$\Rightarrow \boxed{\underbrace{\arctan\left(\frac{-\pi}{4}\right)}_{a} < x < \underbrace{\arctan\left(\frac{\pi}{12}\right)}_{b}}$$

Note: check that $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$!

5. Each of the following questions relates to the equation $x\dfrac{dy}{dx} - \dfrac{3}{\ln x} y = 2x^2 \ln^3 x$.

(a) Is this ODE separable, linear, or other? How do you know?

$\boxed{\text{Linear:}}$ $\quad \dfrac{dy}{dx} - \underbrace{\dfrac{3}{x\ln x}}_{p} y = \underbrace{2x\ln^3 x}_{q}$

(b) Compute the general solution for this ODE.

Hint: Use u-substitution to find the integrating factor $m(x)$; notice that $p(x)$ has a minus sign; and watch your algebra! There's *lots* of cancellation when finding $m(x)$!.

$m(x) = \exp\left(\int -\frac{3}{x\ln x}\right) = \exp\left(-3\ln(\ln x)\right) = \exp\left(\ln (\ln x)^{-3}\right) = \dfrac{1}{\ln^3 x}$

$\Rightarrow$ general solution: $\boxed{y = C\ln^3 x + x^2 \ln^3 x}$

(c) Find the particular solution of this ODE subject to the initial condition $y(\underset{x_0}{e}) = e + e^2$.

$e + e^2 = C\ln^3 e + e^2 \ln^3 e$
$= C + e^2$
$\Rightarrow C = e$

$\boxed{y = e\ln^3 x + x^2 \ln^3 x}$

(d) Determine the interval in which the solution in part (c) exists.

Hint: In precalc, you learned where things like $\ln(x)$, etc., are continuous.

- using the theorem from class, we find where p & q continuous and pick the largest such interval containing x_0.

p: continuous when $\underbrace{x \neq 0}_{\text{frac}}$ & $\underbrace{x > 0}_{\ln} \Rightarrow (0, \infty)$

q: continuous when $x > 0 \Rightarrow (0, \infty)$

intersection is $(0,\infty)$ & x_0 is in there!

$\boxed{\text{Ans: } (0, \infty)}$