2.

(a) True. Isolating y'' yields

$$y'' + \underbrace{\frac{Q(x)}{x^2-4}}_{Q/p} y' + \underbrace{\frac{R(x)}{x^2-4}}_{R/p} y = 0, \quad \left(\begin{matrix} Q \ \& \ R = \\ \text{polys} \end{matrix}\right)$$

and since $p(x_0) = p(2) = 0$, no power series exists for Q/p or R/p @ $x_0 = 2$.

(b) False. $f(t) = t^t$ doesn't.

(c) False. Using partial fractions,

$$F(s) = \frac{1}{2s^2+10s+12} = \frac{1}{2}\left(\frac{1}{s+2}\right) - \frac{1}{2}\left(\frac{1}{s+3}\right)$$

$\Rightarrow$ (by table) $f(t) = \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-3t}$ has $F(s)$ as its Laplace.

(d) False. This only has a chance to be true if $x_0 = 0$ is an ordinary point of the ODE (i.e. if Q/p & R/p have power series about x_0).

ignore this!

(e) False. $f(t)$ and $g(t) = \begin{cases} f(t) & t \neq 5 \\ \boxed{\text{ANYTHING}} & t = 5 \end{cases}$ have same Laplace.

5 isn't special!

Can be literally anything (a #, $\pm\infty$, "DNE"... ANY thing!)

(f) False.

- $\mathcal{L}(f'') = \boxed{s^2\mathcal{L}(f) - sf(0) - f'(0)}$
- $s\mathcal{L}(f') - f(0) = s(s\mathcal{L}(f) - f(0)) - f(0)$
  $= \boxed{s^2\mathcal{L}(f) - sf(0) - f(0).}$

not the same!

(g) False. The intersection of $(0, 5/2)$ an $[-1,2]$ is $(0,2]$, but 2 is singular (since this intersection doesn't contain an open interval around 2).

↳ Note: Because the intersection is $(0,2]$, both Q/P, R/P have power series centered at 2. Their radii of convergence just isn't >0.

(h) False. $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$, not $s\mathcal{L}(f) - f'(0)$.

(i) ~~False. True! (whoops!)~~ False. (double whoops!)

Reindex the first series: $\sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$.

so $\sum_{n=0}^{\infty} (n+1) a_{n+1} x^n = \sum_{n=0}^{\infty} n(n+1) a_n x^n$ $\qquad a_{n+1} = n a_n$ (★)

$\Rightarrow (n+1)a_{n+1} = n(n+1)a_n \Leftarrow$ for all n.

↳ This is False for another reason: In the conclusion, $na_n = n(n+1)a_n \Leftrightarrow na_n = n^2 a_n + na_n \Leftrightarrow 0 = n^2 a_n$ for all n. This is true only if $a_n = 0$! So, if $\{a_n\}$ is a nonzero sequence satisfying (★), the hypothesis is true but the conclusion is false.

see? True/False Questions make us think!

2].

(j) False.

- $\frac{Q}{P} = \frac{Q}{1-x^2}$ has power series at $x \neq \pm 1$ (b/c Q = poly)

- $\frac{R}{P} = \frac{R}{1-x^2}$ is the same (b/c R=poly)

$\Rightarrow$ About $x_0 = 0$, the interval of convergence for $\underbrace{\frac{Q}{P} \text{ is } (-1,1)}_{R.C.(Q/P)=1}$ & same for $\underbrace{\frac{R}{P}}_{R.C.(R/P)=1}$.

Using "the Power series theorem,"

$$R.C.\left(\begin{matrix}\text{power series}\\ \text{sol'n of ODE}\end{matrix}\right) \geq \min\left(R.C.\left(\tfrac{Q}{P}\right), R.C.\left(\tfrac{R}{P}\right)\right)$$
$$= \min(1, 1)$$
$$= 1.$$

So, now such solution can have radius of convergence < 1!