# Exam 2 Preview Solutions

1. General Solution: $x\sin y + x^2 + y^3 = C$

   Particular Solution: $x\sin y + x^2 + y^3 = \pi^3$

2. (a) $\dfrac{N_x - M_y}{M} = \dfrac{y-0}{-4} = -\dfrac{y}{4}$ is a function with only $y$'s. Therefore,

   $$m(y) = \exp\left(-\int \frac{y}{4}\,dy\right) = \exp\left(-\frac{y^2}{8}\right)$$

   is your integrating factor. You should verify that multiplying the given ODE by this integrating factor really *does* give an ODE which is exact.

   (b) Notice that $\dfrac{M_y - N_x}{N} = 4$, and so this is a function with "only $x$'s". Hence,

   $$m(x) = \exp\left(\int 4\,dx\right) = \exp(4x)$$

   is an integrating factor that works.

3. Answer: (vi) and (vii)

   Justification: Here,

   $$f(x,y) = \frac{\ln(\ln(3x)) - \ln x}{\tan y} \quad \text{and} \quad \frac{\partial f}{\partial y}(x,y) = -\frac{\sec^2 y\,(\ln(\ln(3x)) - \ln x)}{\tan^2 y},$$

   and we observe that anything that breaks $f_y$ also breaks $f$. Note that lots of things could break $f$:

   - $\tan y = 0 \iff y = n\pi$ for integer $n$ (yields dividing by 0)
   - $\sin y = 0 \iff y = n\pi$ for integer $n$ (breaks $1/\tan y$)
   - $x \le 0$ (breaks $\ln x$ in numerator)
   - $3x \le 0 \iff x \le 0$ (breaks $\ln(3x)$ in numerator)
   - $\ln(3x) \le 0 \iff 3x \le 1 \iff x \le 1/3$ (breaks $\ln(\ln(3x))$ in numerator)

   **Note:** The above list is a list of things which break $f$ and/or $f_y$; this is *not* a list of things which make $f$ and/or $f_y$ defined/continuous!

   Combining these things, we see that $x \le 1/3$ and $y = n\pi$ for integer $n$ both break $f$ (and $f_y$), so we eliminate all answer choices $(x_0, y_0)$ with $x_0 \le 1/3$ and/or $y_0 = n\pi$.

   The result? (vi) and (vii) are both valid.

4. (a) <u>General Solution</u>: $y = e^{2x}\left(c_1 \sin(7x) + c_2 \cos(7x)\right)$

   <u>Particular Solution</u>: $y = e^{2x}\left(\frac{3}{7}\pi e^{-2\pi}\sin(7x) - \pi e^{-2\pi}\cos(7x)\right)$

   (b) <u>General Solution</u>: $y = c_1 e^{7x/4} + c_2 e^{-x}$

   <u>Particular Solution</u>: $y = \frac{12}{11}e^{7x/4} - \frac{12}{11}e^{-x}$

   (c) <u>General Solution</u>: $y = c_1 e^{-x} + c_2 x e^{-x}$

   <u>Particular Solution</u>: $y = ee^{-x} + exe^{-x}$

5. (a) Repeated root $r_1 = r_2 = 2$ gives a characteristic equation $(r-2)(r-2) = 0 \iff r^2 - 4r + 4 = 0$. Hence, the ODE is $\boxed{y'' - 4y' + 4y = 0}$.

   (b) Complex roots $r_1 = -3 + 2i$ and $r_2 = -3 - 2i$: This gives characteristic equation

$$(r - r_1)(r - r_2) = 0 \iff r^2 + 6r + 13 = 0.$$

   Thus, we have the corresponding ODE $\boxed{y'' + 6y' + 13y = 0}$.

   (c) Real non-repeated roots $r_1 = -5$ and $r_2 = 5$ yield a characteristic equation $(r + 5)(r - 5) = 0$, i.e. $r^2 - 25 = 0$. This corresponds to the ODE $\boxed{y'' - 25y = 0}$.

6. (a) You can verify that

$$y_1' = e^x \sin(3x) + 3e^x \cos(3x), \quad y_1'' = 6e^x \cos(3x) - 8e^x \sin(3x),$$

$$y_2' = e^x \cos(3x) - 3e^x \sin(3x), \quad \text{and} \quad y_2'' = -6e^x \sin(3x) - 8e^x \cos(3x).$$

   From here, just plug and chug to show that both $y_1'' - 2y_1' + 10y_1 = 0$ and $y_2'' - 2y_2' + 10y_2 = 0$ hold.

   (b) $W(y_1, y_2) = -3e^{2x}$.

   (c) They <u>do</u> form a fundamental system of solutions: By (a), both $y_1$ and $y_2$ solve that ODE, and by (b), $W(y_1, y_2) \neq 0$. Thus, $y_1$ and $y_2$ satisfy both conditions of the definition.

   (d) This is <u>false</u>: Because $y_1$ and $y_2$ form a fundamental set of solutions, *every* solution $y_3$ of the ODE $y'' - 2y' + 10y = 0$ has the form $y_3 = c_1 y_1 + c_2 y_2$ for some choice of constants $c_1$ and $c_2$.