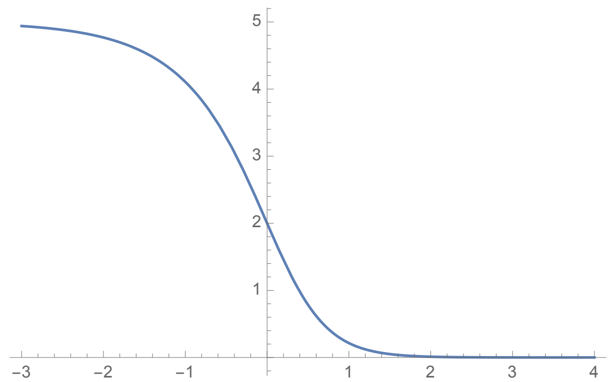
(a) $y = 0$, $y = 5$, and $y = 9$

(b) $y = 0$ is **stable**; $y = 5$ is **unstable**; $y = 9$ is **stable**.

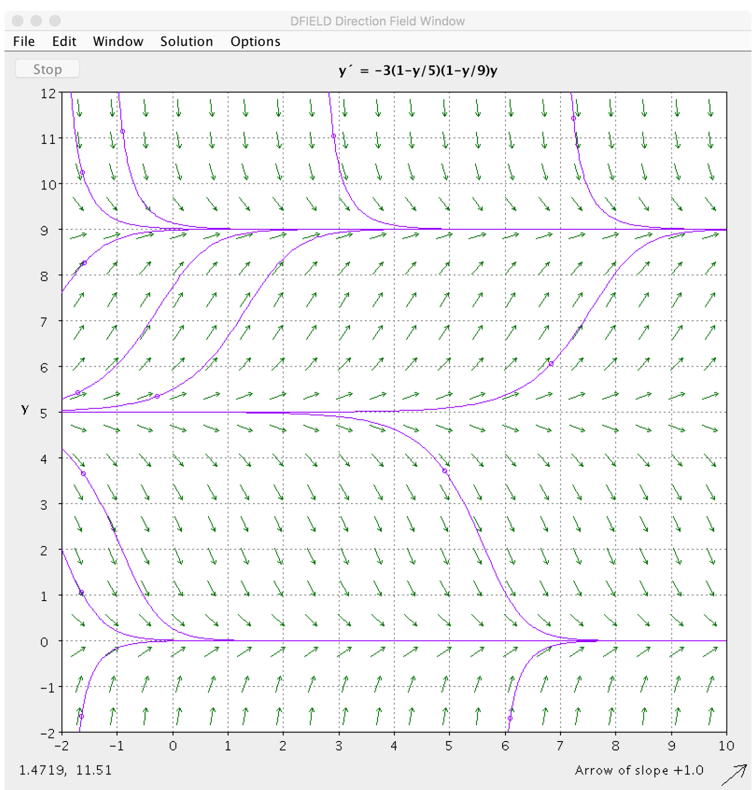
(c) Given that $y(0) = 2$, $\lim\limits_{x\to\infty} y(x) = 0$.

- To see this, note first that $y = 2$ lives in the y-interval $(0, 5)$.
- Then, because $y = 0$ is asymptotically stable, every integral curve in the y-intervals $(-\infty, 0)$ and $(0, 5)$ go to 0 as $x \to \infty$.
- The graph of the **actual** solution (which you were told not to find) verifies our claim:



(d) (iv)

(e) I cheated and used the computer; **you** should do it without cheating, and upon doing so, your result should look something like this:



(f) False

(g) (ii)

(h) This equation (like **every** autonomous ODE) is separable:

$$\frac{dy}{\left(1-\frac{y}{5}\right)\left(1-\frac{y}{9}\right)y} = -3\,dx.$$

Using partial fractions, we can decompose the lefthand side:

$$\left(\frac{1}{y} - \frac{9}{4(y-5)} + \frac{5}{4(y-9)}\right)dy = -3\,dx.$$

Now, we integrate each side:

$$\int \left(\frac{1}{y} - \frac{9}{4(y-5)} + \frac{5}{4(y-9)}\right)dy = \int -3\,dx \iff \ln|y| - \frac{9}{4}\ln|y-5| + \frac{5}{4}\ln|y-9| = -3x + C.$$

Next, we rewrite the lefthand side using properties of logarithms:

$$\ln\left(\frac{|y|\,|y-9|^{5/4}}{|y-5|^{9/4}}\right) = -3x + C.$$

Finally, we eliminate the ln, giving a solution in implicit form:

$$\boxed{\frac{|y|\,|y-9|^{5/4}}{|y-5|^{9/4}} = e^{-3x+C}.}$$

Note: We have to leave the absolute values here because we were told initially to allow $y < 0$.

(i) After part (h) above, all that's left is to find the constant C corresponding to the point $(-1, \pi)$. Doing so yields

$$\frac{|\pi|\,|\pi-9|^{5/4}}{|\pi-5|^{9/4}} = e^{-3(-1)+C} \iff C = \ln\left(\frac{\pi(9-\pi)^{5/4}}{e^3(5-\pi)^{9/4}}\right).$$

Note 1: The last step used the property $e^{a+b} = e^a e^b$.

Note 2: We were able to remove the absolute values by rewriting accordingly. In the numerator of C, for example, $\pi - 9 < 0$ implies that $|\pi - 9| = -(\pi - 9)$ (since absolute value makes negative things positive), and because

$$-(\pi - 9) = 9 - \pi,$$

we have that $|\pi - 9| = 9 - \pi$ and hence that $|\pi - 9|^{5/4} = (9-\pi)^{5/4}$. The same is true for the $|\pi - 5|^{9/4}$ term in the denominator.