

**Directions:** Answer each of the following questions. Make sure to read the instructions for each question as you proceed. *For multiple choice questions, indicate your choice(s) by circling/drawing a box around the appropriate selection(s).*

Throughout, consider the transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  defined by  $T : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} -x_2 \\ 0 \\ x_1 \\ x_1 + x_3 \end{pmatrix}$ .

1. **True or False:**  $T$  is a linear transformation. Justify your claim.

2. Compute:

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \underline{\hspace{10cm}}$$

$$T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \underline{\hspace{10cm}}$$

$$T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \underline{\hspace{10cm}}$$

3. Find the canonical matrix  $A$  corresponding to the transformation  $T$  such that  $T(\mathbf{x}) = A\mathbf{x}$  for all  $\mathbf{x}$  or state that no such matrix exists.

4. What is the domain of  $T$ ?

5. What is the codomain of  $T$ ?

6. Find/describe the range of  $T$ .

**Hint:** You can look at the right-hand side of  $T$  and write a *parametric vector form* for  $T$ ; this will suffice!

7. Is the codomain of  $T$  equal to the range of  $T$ ? How do you know? If they *aren't* the same, find a point in  $\text{codomain}(T)$  that *isn't* in  $\text{range}(T)$ .

8. Is  $T$  injective/one-to-one? Justify your claim.

9. Is  $T$  surjective/onto? Justify your claim.

Scratch Paper