

Directions: Answer each of the following questions. Make sure to read the instructions for each question as you proceed. For multiple choice questions, indicate your choice(s) by circling/drawing a box around the appropriate selection(s).

Throughout, let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix}_{2 \times 3} \quad B = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}_{2 \times 2} \quad C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{2 \times 2} \quad v = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}_{3 \times 1}$$

2 pts ea.

1. Compute each of the following or state that it does not exist. If it does not exist, please indicate why!

(a) $A + C$

DNE \leftarrow A & C aren't the same size.

(b) $B - 2C$

$$B - 2C = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} - 2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 - 2(0) & 2 - 2(1) \\ 1 - 2(1) & 3 - 2(0) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -1 & 3 \end{pmatrix}$$

(c) AB

DNE: $\# \text{cols}(A) = 3 \neq 2 = \# \text{rows}(B)$

(d) BA

$$BA = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1(1) + 2(0) & -1(2) + 2(1) & -1(3) + 2(1) \\ 1(1) + 3(0) & 1(2) + 3(1) & 1(3) + 3(1) \end{pmatrix}$$

(e) vA

DNE: $\# \text{cols}(v) = 1$

$\neq 2 = \# \text{rows}(A)$

(f) Av

$$Av = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1(1) + 2(-1) + 3(3) \\ 0(1) + 1(-1) + 1(3) \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

2. Which of the following matrices are in Row Echelon Form (REF)? **Hint:** There may be more than one but there may also be none!

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 \\ 2 & 2 \\ 0 & -5 \end{pmatrix}$

(e) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

↑ pts for either

(c) $\begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3. Which of the following matrices are in Reduced Row Echelon Form (RREF)? **Hint:** There may be more than one but there may also be none!

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 \\ 2 & 2 \\ 0 & -5 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4. Each of the following matrices needs only **one** elementary row operation to be in RREF.

Indicate which operation is required and show the RREF matrix that results from performing that operation.

$$(a) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & 1 \end{pmatrix} \xrightarrow{R_2 = \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & \frac{1}{2} \end{pmatrix}$$

$$(c) \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \end{pmatrix} \xrightarrow{R_2 = R_1 + R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

Scratch Paper