To clarify something from yesterday's lecture:

- I wrote "Find $B\vec{z}$ for $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix}$ and $\vec{z} = \langle 2, -1, -1 \rangle$."
  - ↳ This isn't possible: $B = 3\times 3$ & $\vec{z} = 1 \times 3 \Rightarrow$ can't be multiplied!
  - What I said at the time: "This is the same as doing $B \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ ..." and "I could have written $\vec{z}B$ instead!" (b/c $(1\times 3)\times(3\times 3) \to 1\times 3$).
  - What I meant to say:
    - This is a typo & I should've written $\vec{z} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ instead.
    - You <u>can't</u> reverse the order / "transpose" matrices / vectors when multiplying!
    - $B \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2+1-3 \\ 0-1-1 \\ -4-1-4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -9 \end{pmatrix}$ <u>is not</u> the same as

$$(2,-1,-1)\,B = (2+0+2,\ -2-1-1,\ 6-1-4) = (4,-4,1).$$

I'm sorry for any confusion this may have caused!

<u>Ex</u>: Do AB, BI, CD, DC

From 1^{st} day handout:

$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -3 & 3 \end{pmatrix}$ $D = \begin{pmatrix} 2 & 4 \\ -1 & 1 \\ 0 & 0 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

§1.1 & 1.2 : Elementary row operations + REF/RREF

- See handout for definition of elementary row operations + REF + RREF.

- The goal is to use elementary row operations to put matrix in REF & RREF:

Ex:
$$\begin{pmatrix} \boxed{2} & -1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 6 & -3 & 9 & 3 \\ 0 & 0 & 4 & 1 \end{pmatrix} \xrightarrow[\substack{R2 = 2R2 - R1 \\ R3 = R3 - 3R1 \\ R4 = R4}]{R1 \text{ stays same}} \begin{pmatrix} \boxed{2} & -1 & 3 & 1 \\ 0 & \boxed{1} & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 \end{pmatrix}$$

- Start w/ (1,1)-entry "leading entry in row 1"

Plan ~~to~~ zero out others using operation 3.

all other entries in this column are zero.

now, we focus on leading nonzero entry in row 2.

want to zero these out using operation 3, but they're already done ✓

(rewrite the same matrix)

$$\begin{pmatrix} 2 & -1 & 3 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 \end{pmatrix} \xrightarrow{R3 \leftrightarrow R4} \begin{pmatrix} 2 & -1 & 3 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & \boxed{4} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

row 3 ≡ 0's ; we move it to bottom

this is the leading entry in R3.

we want to zero out this, but it's done!

$$\boxed{\text{This matrix is in REF! (but not RREF...)}}$$

(Cont'd)

$$\begin{pmatrix} \boxed{2} & -1 & 3 & 1 \\ 0 & \boxed{1} & -1 & 1 \\ 0 & 0 & \boxed{4} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

these are the leading entries.

- To get RREF, first make all leading entries equal 1.

$$\xrightarrow[\substack{R1 = \frac{1}{2}R1 \\ R2 = R2 \\ R3 = \frac{1}{4}R3}]{} \begin{pmatrix} \boxed{1} & -1/2 & 3/2 & \frac{1}{2} \\ 0 & \boxed{1} & -1 & 1 \\ 0 & 0 & \boxed{1} & 1/4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

start here

- Then, zero out the entries above leading 1's:

$$\xrightarrow[\substack{R2 = R2 + R3 \\ R1 = R1 - 3/2 R3}]{} \begin{pmatrix} \boxed{1} & -1/2 & 0 & 1/8 \\ 0 & \boxed{1} & 0 & 5/4 \\ 0 & 0 & \boxed{1} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\frac{1}{2} - \frac{3}{2}\left(\frac{1}{4}\right) = \frac{1}{2} - \frac{3}{8} = \frac{1}{8}$

Done w/ these

Do this next

All equal 1

$$\xrightarrow[R_1 = R_1 + \frac{1}{2}R_2]{} \begin{pmatrix} \boxed{1} & 0 & 0 & 3/4 \\ 0 & \boxed{1} & 0 & 5/4 \\ 0 & 0 & \boxed{1} & 1/4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\frac{1}{8} + \frac{1}{2}\left(\frac{5}{4}\right) = \frac{1}{8} + \frac{5}{8} = \frac{6}{8} = 3/4$

All zero'd out

This is in RREF!

Ex: If $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix}$,

(a) Put in REF & (b) put in RREF.

(a)
$$\begin{pmatrix} \boxed{1} & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix} \xrightarrow{R3=R3+2R1} \begin{pmatrix} \boxed{1} & -1 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & -1 & 10 \end{pmatrix}$$

$$\xrightarrow[R3=R3+R2]{} \begin{pmatrix} \boxed{1} & -1 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & \boxed{11} \end{pmatrix} \leftarrow \text{REF!}$$

(b) Now, zero out entries above leading entries: & scale leading entries as needed

$$\begin{pmatrix} \boxed{1} & -1 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & \boxed{11} \end{pmatrix} \xrightarrow[R3=\frac{1}{11}R3]{} \begin{pmatrix} \boxed{1} & -1 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & \boxed{1} \end{pmatrix}$$

$$\xrightarrow[\substack{R_1=R_1-3R3 \\ R_2=R_2-R3}]{} \begin{pmatrix} \boxed{1} & -1 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \xrightarrow[R1=R1+R2]{} \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix}$$

RREF!