Recall: • A matrix is a rectangular array of numbers.

Ask
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• A matrix A is $m \times n$ if it has m rows & n columns:

$$A = \underbrace{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}}_{n \text{ columns}} \Big] \, m \text{ rows}$$

matrices of same size form a group

• $A+B$ is defined if A & B are matrices of same size: $\#\text{rows}(A) = \#\text{rows}(B)$ and $\#\text{cols}(A) = \#\text{cols}(B)$.

↳ If defined, $A+B$ is matrix of element-wise addition:

Ex: $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & -9 \end{pmatrix}$

$$\Rightarrow A+B = \begin{pmatrix} 1+-1 & 2+2 & 3+-3 \\ 4+4 & 5+-5 & 6+6 \\ 7+-7 & 8+8 & 9+-9 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 0 \\ 8 & 0 & 12 \\ 0 & 16 & 0 \end{pmatrix}$$

• AB is defined if $\#\text{cols}(A) = \#\text{rows}(B)$, i.e.: If A is $m \times n$, then AB is defined if & only if B is $n \times k$.

collection of all such matrices form a ring.

$$\underbrace{(m \times n)}_{A} \underbrace{(n \times k)}_{B} \longmapsto \underbrace{m \times k}_{AB}$$

same!

Today we'll learn how to multiply!

• Even if both are defined, AB may not equal BA!

Ex: If $A = 2 \times 3$ & $B = 3 \times 2$, then $AB = 2 \times 2$ but $BA = 3 \times 3$!

All covered yesterday

"New" Stuff

Recall: • A vector is a matrix with one row (or one column).

Ex: $\vec{v} = \langle 1, 2 \rangle$ is a vector in $\mathbb{R}^2$

$\vec{w} = \langle -1, 3, 4 \rangle$ is a vector in $\mathbb{R}^3$

$\vec{x} = \langle x_1, \dots, x_n \rangle$ is a vector in $\mathbb{R}^n$.

• If $\underset{\langle u_1,\dots,u_n\rangle}{\vec{u}}$ & $\underset{\langle v_1,\dots,v_n\rangle}{\vec{v}}$ are vectors of same length, then the **dot product** $\vec{u} \cdot \vec{v}$ is the number

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

Ex: ① Find dot product of $\vec{u} = \langle 1, 2 \rangle$ & $\vec{v} = \langle -3, 4 \rangle$

$\vec{u} \cdot \vec{v} = 1(-3) + 2(4) = -3+8 = 5$

② " " " " $\vec{x} = \langle 1, 1, 5, 6 \rangle$ & $\vec{y} = \langle 0, 1, 1, 0 \rangle$

$\vec{x} \cdot \vec{y} = 1(0) + 1(1) + 5(1) + 6(0) = 6$

• Now, if A is $m \times n$ & B is $n \times k$, then AB is the $m \times k$ matrix whose $(i,j)^{th}$ entry is (row i of A)$\cdot$(col j of B)

Ex: $$\underbrace{\begin{array}{c} R_1 \to \\ R_2 \to \end{array}\overbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}^{A}}_{2\times 2} \underbrace{\begin{array}{ccc} C_1 & C_2 & C_3 \\ \begin{pmatrix} -1 & 0 & -3 \\ 1 & 1 & 1 \end{pmatrix} \end{array}}_{2\times 3 \;\; B} = \begin{pmatrix} R_1\cdot C_1 & R_1\cdot C_2 & R_1\cdot C_3 \\ R_2\cdot C_1 & R_2\cdot C_2 & R_2\cdot C_3 \end{pmatrix}$$

you can imagine "pouring" the rows of A over the columns of B

$$= \begin{pmatrix} 1(-1)+2(1) & 1(0)+2(1) & 1(-3)+2(1) \\ 3(-1)+4(1) & 3(0)+4(1) & 3(-3)+4(1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & -5 \end{pmatrix}$$

Ex: $\boxed{1(d)}$ If $C = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -3 & 3 \end{pmatrix}$ & $D = \begin{pmatrix} 2 & 4 \\ -1 & 1 \\ 0 & 0 \end{pmatrix}$,

2×3 $\quad$ 3×2

Find CD!

$\hookrightarrow$ $CD = \begin{pmatrix} 2+-1+0 & 4+1+0 \\ -2+3+0 & -4+-3+0 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 1 & -7 \end{pmatrix}$

$\boxed{2 \times 2}$

$\boxed{\#1(i)}$

Ex: $\overbrace{B(\vec{u}+\vec{w})}^{\text{parens first!}}$ where $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix}$, $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

$$= \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix} \overbrace{\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \right)}^{\text{can add!}} = \underset{3 \times 3}{\begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix}} \underset{3 \times 1}{\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}}$$

$$= \begin{pmatrix} 2+1-3 \\ 0-1-1 \\ -4-1-4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -9 \end{pmatrix}$$

$\boxed{3 \times 1}$

Note: You cannot multiply two vectors to get a vector! $\Rightarrow$ space of vectors is not a ring

$\hookrightarrow$
- Dot product only gives scalar (#)
- Cross product only defined in dim = 3, 7
- wedge product can only be defined as a vector if used on n vectors of length n+1.