Name: ______________________________

MAS 3105 — Homework 3

Directions: Complete the following problems for a homework grade, being sure to adhere to the Homework Policy on the Homework tab of the course webpage

`http://www.math.fsu.edu/~cstover/teaching/sp18_mas3105/`.

Date Due: Tuesday, April 3.

1. Go to the **#hw3** channel in our course's SLACK room (see course homepage for the URL) and
   (i) post ≥ 1 thing about this homework; and
   (ii) reply to ≥ 1 of your classmates' posts.

   Note: Yes, you will get graded for this question. ☺

2. For each of the following matrices A,
   (i) put A into RREF;
   (ii) find a basis for $\text{col}(\mathsf{A})$;
   (iii) compute $\dim(\text{col}(\mathsf{A}))$;
   (iv) find a basis for $\text{row}(\mathsf{A})$;
   (v) compute $\dim(\text{row}(\mathsf{A}))$;
   (vi) find a basis for $\text{nul}(\mathsf{A})$;
   (vii) compute $\dim(\text{nul}(\mathsf{A}))$; and
   (viii) verify the rank-nullity theorem for A.

(a) $\mathsf{A} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 3 & 1 \\ 3 & -7 & -3 & 7 \end{pmatrix}$

(b) $\mathsf{A} = \begin{pmatrix} -2 & 1 & 3 \\ 4 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & -2 & 4 \end{pmatrix}$

(c) $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & -2 & 3 & -4 \\ 9 & 10 & -11 & 12 \end{pmatrix}$

3. For each of the matrices A in Problem 2, repeat parts (i)–(viii) (of problem 2) for the matrix A^{T}.

4. For each of the following linear transformations T,
    (i) find the domain and codomain of T;
    (ii) find the canonical matrix A for T;
    (iii) find range(T);
    (iv) find the dimension of range(T);
    (v) find ker(T); and
    (vi) find the dimension of ker(T);

   Assume that x_1, x_2, x_3, and x_4 are all real numbers.

   (a) $\mathrm{T} : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 \\ x_1 - x_2 \\ x_2 \\ x_2 - x_1 \\ 2x_1 + 3x_2 \end{pmatrix}$

   (b) $\mathrm{T} : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 - x_4 \\ x_2 - x_3 \end{pmatrix}$

   (c) $\mathrm{T} : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \longmapsto \begin{pmatrix} x_4 \\ -x_1 \\ x_2 - x_3 - x_4 \\ x_1 - x_2 - x_3 \end{pmatrix}$

5. For each of the linear transformations in Problem 4, repeat parts (i)–(vi) (of problem 4) for the linear transformation $S(\mathbf{x}) = \mathsf{A}^{\mathsf{T}}\mathbf{x}$ (where A is the canonical matrix of the transformation in question).

6. Let $\mathsf{A} = \begin{pmatrix} 7 & -9 & -4 & 5 & 3 & -3 & -7 \\ -4 & 6 & 7 & -2 & -6 & -5 & 5 \\ 5 & -7 & -6 & 5 & -6 & 2 & 8 \\ -3 & 5 & 8 & -1 & -7 & -4 & 8 \\ 6 & -8 & -5 & 4 & 4 & 9 & 3 \end{pmatrix}$.

(a) Construct a matrix C whose columns are a basis for $\mathrm{col}(\mathsf{A})$.

(b) Construct a matrix N whose columns are a basis for $\mathrm{nul}(\mathsf{A})$.

(c) Construct a matrix R whose rows are a basis for $\mathrm{row}(\mathsf{A})$.

(d) Construct a matrix M whose columns are a basis for $\mathrm{nul}(\mathsf{A}^\mathsf{T})$.

(e) Form the augmented matrix $\mathsf{S} = \left(\mathsf{R}^\mathsf{T} \,\middle|\, \mathsf{N}\right)$. What are its dimensions?

(f) Find the determinant for S and the inverse S^{-1}, if they exist. If not, state why.

(g) Form the augmented matrix $\mathsf{T} = \left(\mathsf{C} \,\middle|\, \mathsf{M}\right)$. What are its dimensions?

(h) Find the determinant for T and the inverse T^{-1}, if they exist. If not, state why.