Name: ____________________

MAS 3105 — Homework 2

Directions: Complete the following problems for a homework grade, being sure to adhere to the Homework Policy on the Homework tab of the course webpage

`http://www.math.fsu.edu/~cstover/teaching/sp18_mas3105/`.

Date Due: Thursday, March 1.

1. Go to the **#springbreak** channel in our course's SLACK room (see course homepage for the URL) and say something about spring break. Do you like it? Do you not like it? Are you going somewhere? What do you plan to do? Tell us things! Note: Yes, you will get graded for this question. ☺

2. For each of the transformations T in (a)—(i),
   (i) determine whether T is linear;
   (ii) find the domain, the codomain, and the range;
   (iii) compute $T(\mathbf{e}_1), \ldots, T(\mathbf{e}_n)$ where $\mathbf{e}_1, \ldots, \mathbf{e}_n$ are the columns of the $n \times n$ identity matrix I_n;
   (iv) find the canonical matrix A corresponding to T, if it exists;
   (v) determine if T is one-to-one/injective; and
   (vi) determine if T is onto/surjective.

   Assume that w, x, y, and z are real numbers.

   (a) $T : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} y \\ x \end{pmatrix}$

   (b) $T : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x \\ x \end{pmatrix}$

   (c) $T : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x+y \\ x \end{pmatrix}$

   (d) $T : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} 0 \\ y \end{pmatrix}$

   (e) $T : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \end{pmatrix}$

   (f) $T : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

   (g) $T : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$

   (h) $T : \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}$

   (i) $T : \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} w+x+y+z \end{pmatrix}$

3. Let S and T be the linear transformations indicated below. Find the canonical matrix corresponding to the composition $T \circ S$.

(a) $S : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$; $T : \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \longmapsto \begin{pmatrix} y_2 \\ y_1 \end{pmatrix}$

(b) $S : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$; $T : \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \longmapsto \begin{pmatrix} y_1 \\ y_2 \\ y_1 + 2y_2 \end{pmatrix}$

(c) $S : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_1 + x_2 + x_3 \\ x_1 \end{pmatrix}$; $T : \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \longmapsto \begin{pmatrix} y_1 \\ y_2 - y_3 - y_4 \end{pmatrix}$

4. For each of the following matrices,
   (i) find A^3 or state that it doesn't exist,
   (ii) find A^T,
   (iii) find $\det(\mathsf{A})$, and
   (iv) find A^{-1} or state that it doesn't exist.

<u>Justify your claim(s).</u>

(a) $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

(b) $\mathsf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$

(c) $\mathsf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\mathsf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(e) $\mathsf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(f) $\mathsf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

(g) $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$

(h) $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 11 \end{pmatrix}$

(i) $\mathsf{A} = \begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 4 & 4 & -2 \\ -2 & 0 & 1 & 3 \\ 3 & 2 & 1 & -5 \end{pmatrix}$

5. Let $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Write **true** or **false** for each of the following statements about A.

Hint: You may not need to work super-hard for most of these!

(a) $\mathsf{A}\mathbf{x} = \mathbf{0}$ has non-trivial solutions.

(b) The RREF of A is equal to the 3×3 identity matrix I_3.

(c) The columns of A form a linearly dependent set.

(d) The transformation $\mathbf{x} \mapsto \mathsf{A}\mathbf{x}$ is one-to-one.

(e) The transformation $\mathbf{x} \mapsto \mathsf{A}\mathbf{x}$ is onto.

(f) The columns of A span $\mathbb{R}^3$.

(g) The range of the transformation $\mathbf{x} \mapsto \mathsf{A}\mathbf{x}$ equals its codomain.

(h) The matrix A^{-1} exists.

(i) $\det(\mathsf{A}) = 7$.

(j) There exists some $\mathbf{b} \in \mathbb{R}^3$ for which $\mathsf{A}\mathbf{x} = \mathbf{b}$ <u>doesn't</u> have a solution.

(k) A^T is invertible.