Name: ______________________

MAS 3105 — Homework 1

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems **must** be done on scratch paper and not on this handout!* **Date Due:** Thursday, February 1.

1. **Note**: Yes, you will get graded for this question. ☺

   (a) Navigate to our course homepage at

   `http://www.math.fsu.edu/~cstover/teaching/sp18_mas3105/`

   (b) Read and familiarize yourself with the three resources listed under *Supplementary Resources* on the General Info tab.

   (c) Follow the instructions for using Slack messenger.

   Note: This may require that I approve your email address, so to avoid some last minute glitch where I don't get to your approval on-time, please don't wait to do this!

   (d) Navigate to the channel `#introductions` in the left column under Channels; its browser URL should be something like `https://spring2018-linalg.slack.com/messages/C8WQS8E77/`.

   (e) Introduce yourself by answering each of the following questions:

   Note: This will be visible to everyone who signs into our class's chat room, so you definitely want to keep these answers PG-13, safe for work, and non-incriminatory. ☺

   (i) What is your name?

   (ii) Where are you from (using any interpretation you'd like)?

   (iii) What is the best (interpret this however you'd like) place you've ever visited/lived? Why is it so special to you?

   (iv) How long have you been in Tallahassee?

   (v) What is your major?

   (vi) What do you like to do for fun? (besides linear algebra, of course!)

   (vii) What is the coolest math/science "thing" you know? Why is it interesting to you?

   (f) Under which username did you register for Slack? ______________________

2. Put each of the following matrices in reduced row echelon form (RREF).

   Note: If you get to row echelon form (REF) *before* you get to RREF, please indicate the REF matrix row equivalent to the one you started with!

   (a) $\mathsf{A} = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$

   (b) $\mathsf{B} = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -1 \end{pmatrix}$

   (c) $\mathsf{C} = \begin{pmatrix} -3 & 6 & 9 \\ 1 & -2 & -3 \\ 0 & 5 & 1 \\ 0 & 0 & -8 \end{pmatrix}$

   (d) $\mathsf{D} = \begin{pmatrix} 2 & 1 & 0 & -1 \\ -1 & 1 & 1 & 3 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

3. For each of the following linear systems, decide whether it's (i) consistent with a unique-solution, (ii) consistent with a non-unique solution, or (iii) inconsistent. If a system is consistent, solve it using matrices; otherwise, state why it isn't and/or how you know it's not.

(a) $$\begin{array}{rcrcl} x_1 & + & x_2 & = & 3 \\ 2x_1 & - & x_2 & = & 1 \end{array}$$

(b) $$\begin{array}{rcrcrcl} x_1 & + & & & x_3 & = & 3 \\ 2x_1 & - & x_2 & & & = & 1 \end{array}$$

(c) $$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & = & 1 \\ -x_1 & - & x_2 & + & x_3 & = & 1 \\ x_1 & & & & & = & 2 \end{array}$$

(d) $$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & = & 1 \\ -x_1 & - & x_2 & + & x_3 & = & 1 \\ & & & & x_3 & = & 0 \end{array}$$

(e) $$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 1 \\ -x_1 & - & x_2 & + & x_3 & = & -1 \\ & & & & x_3 & = & 0 \end{array}$$

(f) $$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & + & 2x_3 & = & 7 \\ 4x_1 & - & 4x_2 & + & x_3 & = & -1 \\ x_1 & - & 3x_2 & + & x_3 & = & 1 \end{array}$$

4. For each of the six systems of equations in exercise 3, write a vector equation that is equivalent.

5. For each of the six systems of equations in exercise 3, write a matrix equation that is equivalent.

6. Each of the following matrices corresponds to the **augmented matrix** of a linear system. Determine whether the corresponding system is (i) consistent with a unique-solution, (ii) consistent with a non-unique solution, or (iii) inconsistent. **Justify your claim!**

(a) $\mathsf{A} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \end{pmatrix}$

(b) $\mathsf{B} = \begin{pmatrix} 0 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\mathsf{C} = \begin{pmatrix} 0 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(d) $\mathsf{D} = \begin{pmatrix} 0 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \\ -2 & 1 & 3 & 11 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(e) $\mathsf{E} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & -8 \\ -9 & 10 & -11 & 12 \end{pmatrix}$

7. Is $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$ a linear combination of the vectors $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{a}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, and $\mathbf{a}_3 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$? Why or why not?

8. Is $\mathbf{b} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$ a linear combination of the vectors formed by the columns of $\mathsf{A} = \begin{pmatrix} -1 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & -4 \end{pmatrix}$? Why or why not?

9. **True or False:** Every vector in $\mathbb{R}^2$ is a linear combination of the vectors $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$. **Justify your claim!**

   Hint: The clearest way to do this is to start with an arbitrary vector $\mathbf{w} = \begin{pmatrix} a \\ b \end{pmatrix}$ in $\mathbb{R}^2$ and to either write $\mathbf{w} = s\mathbf{u} + t\mathbf{v}$ for real numbers s and t <u>or</u> to show that no such expression is possible.

10. Describe all solutions of $\mathsf{A}\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

    (a) $\begin{pmatrix} 1 & 3 & -3 & 4 \\ 0 & 1 & -4 & 5 \end{pmatrix}$

    (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 34 \end{pmatrix}$

    (c) $\begin{pmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

11. Let

$$\mathsf{A} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & 6 \\ 4 & 2 & -3 \end{pmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix},$$

write $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ for the columns of A, and let $W = \text{span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

(a) Write the augmented matrix for the linear system that corresponds to the matrix equation $\mathsf{A}\mathbf{x} = \mathbf{b}$.

(b) Solve the matrix equation from (a) and write the solution as a vector.

(c) Is $\mathbf{b}$ in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

(d) How many vectors are in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

(e) Is $\mathbf{b}$ in W?

(f) How many vectors are in W?

(g) Show that $\mathbf{a}_2$ is in W.

(h) Show that $\mathbf{a}_1 - \mathbf{a}_3$ is in W.

(i) Do the columns of A form a linearly independent set? Why or why not?

(j) Let h be a real number and let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ h \end{pmatrix}$. For which value(s) h is $\mathbf{u}$ in $\text{span}\{\mathbf{a}_1, \mathbf{a}_2\}$?

(k) Let $\mathbf{u}$ be as in (j). For what value(s) of h is $\{\mathbf{a}_2, \mathbf{b}, \mathbf{u}\}$ linearly *dependent*?