T / F.

(a) False. (REF may not be unique)

(b) False. (may have ∞-many)

(c) True.

(d) False (may have no sol'n)

(e) True.

(f)* True. (This is hard but: $A\vec{x} = \vec{b}$ has > 1 solution if the line/plane/whatever containing those solutions passes through some point $\vec{p}$, and the solution set to $A\vec{x} = \vec{0}$ is just the translate of that, parallel to it, such that $\vec{p} = \vec{0}$. See pp 46–47.)

(g)* False. (true if $A\vec{x} = \vec{b}$ has sol'n for all $\vec{b}$. for counter-ex. to written question: If $\vec{b} = \vec{0}$ is "some $\vec{b}$" & $A\vec{x} = \vec{0}$ has a non-trivial sol'n, then columns of A span space w/ fewer dimensions!)

(h) False. (RREF may have $[0, \dots, 0, 1]$ row)

(i) True. (RREF unique)

(j) False. ($A\vec{x} = \vec{0}$ always has trivial solution! They omitted the word "only" here....)

(k) – (n) → skipped.

(o) True.* (see (f) & note: going $\vec{b} \to \vec{c}$ is going $\vec{b} \to \vec{0} \to \vec{c}$, so $A\vec{x} = \vec{c}$ has solution set parallel to that of $A\vec{x} = \vec{b}$. This is a bad question.)

17)

(p) True. ($A = m \times n$ w/ cols spanning $\mathbb{R}^m \Rightarrow A\vec{x} = \vec{b}$ always has a sol'n; A r.e. $B \Rightarrow B\vec{x} = \vec{b}$ always does, too, so cols(B) span $\mathbb{R}^m$.)

(q) False. ($\vec{v}_3 = \vec{v}_1 + \vec{v}_2 \Rightarrow$ L.D., for example)

(r) True. (3 vecs L.I. $\Rightarrow$ vecs have ≥ 3 components).

(s) False.

(t) True. ($-\vec{u} = -1\vec{u} + 0\vec{v}$).

(u) False. (need $\vec{u}$ &. $\vec{v}$ to be L.I.)

(v) True. ($\vec{w}$ linear combo $\Rightarrow \{\vec{u}, \vec{v}, \vec{w}\}$ L.D. $\Rightarrow$ $\vec{u}$ Linear Combo.

OR: If $\vec{w} = x_1\vec{u} + x_2\vec{v}$, then $\vec{u} = \frac{1}{x_1}(\vec{w} - x_2\vec{v})$.)

(w) True. ($\vec{v}_2$ not a multiple of $\vec{v}_1 \Rightarrow \{\vec{v}_1, \vec{v}_2\}$ L.I. Now, adding $\vec{v}_3$ will stay L.I $\Leftrightarrow \vec{v}_3$ not a linear combo of $\vec{v}_1$ & $\vec{v}_2$ (by defn. of L.I./span)).