SOLUTION TO PRACTICE PROBLEM

$$A = \begin{bmatrix} 36 & 51 & 13 \\ 52 & 34 & 74 \\ 0 & 7 & 1.1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 33 \\ 45 \\ 3 \end{bmatrix}$$

CHAPTER 1 SUPPLEMENTARY EXERCISES

1. Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.

a. Every matrix is row equivalent to a unique matrix in echelon form.

b. Any system of n linear equations in n variables has at most n solutions.

c. If a system of linear equations has two different solutions, it must have infinitely many solutions.

d. If a system of linear equations has no free variables, then it has a unique solution.

e. If an augmented matrix $[\,A \quad \mathbf{b}\,]$ is transformed into $[\,C \quad \mathbf{d}\,]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution sets.

f. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

g. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of A span $\mathbb{R}^m$.

h. If an augmented matrix $[\,A \quad \mathbf{b}\,]$ can be transformed by elementary row operations into reduced echelon form, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

i. If matrices A and B are row equivalent, they have the same reduced echelon form.

j. The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

k. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$, then A has m pivot columns.

l. If an $m \times n$ matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$ in $\mathbb{R}^m$.

m. If an $n \times n$ matrix A has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix.

n. If 3×3 matrices A and B each have three pivot positions, then A can be transformed into B by elementary row operations.

o. If A is an $m \times n$ matrix, if the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.

p. If A and B are row equivalent $m \times n$ matrices and if the columns of A span $\mathbb{R}^m$, then so do the columns of B.

q. If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^3$ is a multiple of one of the other vectors, then S is linearly independent.

r. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are not in $\mathbb{R}^2$.

s. In some cases, it is possible for four vectors to span $\mathbb{R}^5$.

t. If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^m$, then $-\mathbf{u}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$.

u. If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are nonzero vectors in $\mathbb{R}^2$, then $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

v. If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

w. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are in $\mathbb{R}^5$, $\mathbf{v}_2$ is not a multiple of $\mathbf{v}_1$, and $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. Then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

x. A linear transformation is a function.

y. If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.

z. If A is an $m \times n$ matrix with m pivot columns, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is a one-to-one mapping.

2. Let a and b represent real numbers. Describe the possible solution sets of the (linear) equation $ax = b$. [*Hint:* The number of solutions depends upon a and b.]

3. The solutions (x, y, z) of a single linear equation

$$ax + by + cz = d$$

form a plane in $\mathbb{R}^3$ when a, b, and c are not all zero. Construct sets of three linear equations whose graphs (a) intersect in a single line, (b) intersect in a single point, and (c) have no