Exam 1 Preview

Here's a bit of logistical info about the exam.

- There will be 5–7 questions overall, and some will have multiple parts.
- The exam will cover the following textbook sections:
  - §2.1 (the first stuff we did: matrix multiplication, elements of a matrix, etc.)
  - §1.1–1.2 (systems of equations + (R)REF + using (R)REF to solve systems)
  - §1.3 (vector equations, i.e. linear combos + spans)
  - §1.4–1.5 ($\mathsf{A}\mathbf{x} = \mathbf{b}$ and/or $\mathsf{A}\mathbf{x} = \mathbf{0}$)
  - §1.7 (linearly dependent/independent)
- You should expect the following question formats:
  - computation questions (e.g. using matrices to solve systems from start to finish)
  - multiple-choice questions
  - True/False questions (which may or may not require justification).

  The True/False questions will mostly look like those from the textbook (which I include here for those of you without the textbook).

Now, here are some sample questions that you should be able to answer before the exam.

1. Let $\mathsf{A} = \begin{pmatrix} 2 & 0 & -1 & -1 \\ 3 & 1 & 2 & -5 \\ -4 & 0 & 4 & 11 \end{pmatrix}$ be an augmented matrix.

   (a) Put A into Row Echelon Form (REF).

   (b) Put A into Reduced Row Echelon Form (RREF).

   (c) Write a system of linear equations associated to A using x_1, x_2, etc. as your variables.

   (d) Is the system from part (c) consistent? Why or why not?

   (e) Write the solution(s) to the system from part (c) in parametric vector form or state that no solutions exist.

   (f) **True or False:** A is row equivalent to $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Justify your answer!

2. (a) Draw the span of a single nonzero vector in $\mathbb{R}^2$.

   (b) Draw the span of two linearly dependent vectors in $\mathbb{R}^2$.

   (c) Draw the span of two linearly independent vectors in $\mathbb{R}^2$.

   (d) Draw the span of two linearly independent vectors in $\mathbb{R}^3$

   (e) Draw a vector in the span of two linearly independent vectors in $\mathbb{R}^3$.

   (f) Draw a vector linearly independent of two linearly independent vectors in $\mathbb{R}^3$.

3. Let $\mathsf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ -2 & 0 \end{pmatrix}$ and $\mathbf{v} = \langle 2, 1, -3 \rangle$. Provide an example matching each of the following criteria or state that no such example exists. Justify your answer!

   (a) Give an example of a matrix C such that the product BC exists but the product CB does not exist.

   (b) Give an example of a matrix D such that both products BD and DB exist but $\mathsf{BD} \neq \mathsf{DB}$.

   (c) Give an example of a matrix E such that neither BE nor EB exist.

   (d) Give an example of a matrix F such that $\mathsf{FB} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ("the identity matrix"). Is BF equal to "the identity matrix" of the appropriate dimension?

   (e) Give an example of a vector $\mathbf{w}$ such that the dot product $\mathbf{v} \cdot \mathbf{w}$ exists and equals 7.

(f) Give an example of a vector $\mathbf{a}$ such that the vectors $\{\mathbf{a}, \mathbf{v}\}$ are linearly independent.

(g) Give an example of a vector $\mathbf{b}$ such that the vectors $\{\mathbf{v}, \mathbf{b}\}$ are linearly dependent.

(h) Give an example of a vector $\mathbf{k}$ such that the vectors $\{\mathbf{v}, \mathbf{0}, \mathbf{k}\}$ are linearly dependent.

4. Let $\mathsf{A} = \begin{pmatrix} -1 & -2 & -3 \\ 7 & 8 & 9 \\ h & -5 & -6 \end{pmatrix}$, let $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ denote the columns of A, and let $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ denote the rows of A.

(a) For which value(s) h does $\mathsf{A}\mathbf{x} = \mathbf{0}$ have only the trivial solution?

(b) For which value(s) h does $\mathsf{A}\mathbf{x} = \mathbf{0}$ have nontrivial solutions? In this case, express the solutions in terms of one or more free variables.

(c) Let h be as in (a). Give an example of a vector $\mathbf{u}$ for which the set $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{u}\}$ is linearly independent or state that no such vector exists.

(d) Let h be as in (b). Give an example of a vector $\mathbf{u}$ for which the set $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{u}\}$ is linearly independent or state that no such vector exists.

(e) Let h be as in (a). Give an example of a vector $\mathbf{u}$ for which the set $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{u}\}$ is linearly independent or state that no such vector exists.

(f) Let h be as in (b). Give an example of a vector $\mathbf{u}$ for which the set $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{u}\}$ is linearly independent or state that no such vector exists.

5. Practice True/False questions by doing problems 1(a)–1(z) from `Exam1TF.pdf` except for parts (k)–(n) and parts (x)–(z).