Vector Spaces

Recently, I mentioned that I'd begin using the phrase "vector space" but wouldn't formally define it in class. To alleviate the burden caused, I've decided to give you a handout on the topic instead.

Here's the definition

Definition 1: A *vector space* is a nonempty set V of objects (called *vectors*) and a pair of operations—called *addition* (denoted "+") and *scalar multiplication* (denoted by juxtaposition, i.e. $c\mathbf{u}$ for the scalar multiple of $\mathbf{u}$ by c)—which together will always satisfy the following ten axioms (for all scalars c and d and for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V):

These should look like the subspace axioms!

(i) $\mathbf{u} + \mathbf{v} \in V$ } We can add things!

(ii) $c\mathbf{u} \in V$ } We can scalar multiply

(iii) there exists a zero vector $\mathbf{0} \in V$ which satisfies $\mathbf{u} + \mathbf{0} = \mathbf{u}$

(iv) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(v) $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

(vi) for each $\mathbf{u} \in V$, there exists a vector $-\mathbf{u} \in V$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(vii) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

(viii) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

(ix) $c(d\mathbf{u}) = (cd)\mathbf{u}$

(x) $1\mathbf{u} = \mathbf{u}$

Addition and scalar multiplication work the way we want them to! (iii)–(x)

Over the next week or so, my plan is to revisit this handout ≥ 1 time(s) to give examples, details for verifying examples, etc., of vector spaces. Until then, just remember the mantra:

When you see "...n-dimensional vector space V..." just replace it with "...subspace of $\mathbb{R}^n$..."!