Name: _____

MAS 3105 — Matrices I

Directions: Split into groups and (attempt to) answer each of the following questions with your group members. Don't worry about questions you are unable to answer!

In what follows, let

$$\mathsf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \qquad \mathsf{B} = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix} \qquad \mathsf{C} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -3 & 3 \end{pmatrix} \qquad \mathsf{D} = \begin{pmatrix} 2 & 4 \\ -1 & 1 \\ 0 & 0 \end{pmatrix} \qquad \mathsf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and let

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}.$$

1. Determine whether each of the following operations is possible. If it *is* possible, do it; if not, state why it's impossible.

   (a) $\mathsf{A} + \mathsf{B}$

   (b) $\mathsf{B} + \mathsf{I}$

   (c) AB **Hint**: This is the matrix product of A and B.

   (d) CD

   (e) DC

(f) BI

(g) $\mathsf{A}\mathbf{u}$

(h) $2\mathbf{v}$

(i) $\mathsf{B}(\mathbf{u} + \mathbf{w})$

(j) $\mathsf{B}\mathbf{u} + \mathsf{B}\mathbf{w}$

(k) $\mathbf{uw}$

(l) $\mathbf{u} \cdot \mathbf{w}$ **Hint**: This is the dot product of $\mathbf{u}$ and $\mathbf{w}$.

(m) $\det \mathsf{B}$ **Hint**: This is the determinant of B.

(n) C^T **Hint**: This is the transpose of C.

2. (a) Put the matrix B in row echelon form.

(b) Put the matrix B in *reduced row echelon form.*

(c) Using the above, find the inverse of the matrix B if it exists. **Hint**: Using part (m) from the first problem, you should be able to tell whether it exists or not *without* doing any work.

(d) Using part (c), solve the vector equation $\mathsf{B}\mathbf{x} = \mathbf{u}+\mathbf{w}$ for $\mathbf{x}$. **Hint**: B, $\mathbf{u}$, and $\mathbf{w}$ are all known/given above.