Answers:

① Yes

② ∞-many choices of the form:

$$\vec{x} = \begin{pmatrix} 3/5 \\ 1/25 \\ 7/25 \\ 26/25 - 2x_5 \\ x_5 \end{pmatrix}$$

③ True

④ Consistent w/ non-unique solutions.

⑤ The vectors $\vec{x}$ from ② all work, so picking an x_5 would give one particular vector.

Explanations on subsequent pages!

① $\vec{v}$ can be written as a linear combo of $\vec{u}_1, \ldots, \vec{u}_5$ if & only if there is <u>some</u> vector $\vec{x}$ such that

$$A\vec{x} = \vec{v}$$

where $A = [\vec{u}_1 | \cdots | \vec{u}_5]$. This system has augmented matrix $M = [\vec{u}_1 | \cdots | \vec{u}_5 | \vec{v}]$, i.e.

$$M = \begin{pmatrix} -1 & 0 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 & 2 \\ 2 & -2 & 3 & 1 & 2 & 3 \\ 3 & 1 & 4 & 1 & 2 & 4 \end{pmatrix}.$$

We put this in REF:

$$M \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 3/5 \\ 0 & 1 & 0 & 0 & 0 & 1/25 \\ 0 & 0 & 1 & 0 & 0 & 7/25 \\ 0 & 0 & 0 & 1 & 2 & 26/25 \end{pmatrix}$$

This is actually RREF: I cheated and used a computer. :P

<u>HOWEVER</u>: Here is <u>an</u> REF one, not using a computer:

$$\begin{pmatrix} 1 & 2 & 0 & -1 & 2 & 1 \\ 0 & 1 & -2 & -2 & 0 & -1 \\ 0 & 0 & 5 & 8 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 3 \end{pmatrix}$$

~~[Note: variable names may be different than with the computer RREF]~~

Because there is no $[0, 0, \ldots, 0, b]$, $b \neq 0$ row, there is <u>a</u> solution!

② Using eq. ④, x_5 is a free var. and $x_4 = 26/25 - 2x_5$. So, there are ∞-many choices, all of form:

$$x_1 = 3/5$$
$$x_2 = 1/25$$
$$x_3 = 7/25$$
$$x_4 = 26/25 - 2x_5$$
$$x_5 = \text{free}$$

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③. $\vec{v}$ is a linear combo of $\vec{u}_1, \dots, \vec{u}_5$ iff there are $x_1, \dots, x_5$ such that

$$\vec{v} = x_1\vec{u}_1 + \dots + x_5\vec{u}_5 .$$

By ②, such a combo exists, so this is true.

④ By ②, consistent w/ non-unique solutions.

⑤ The vector $\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix}$ w/ values from ②