Mar 8, 2018

Exam 2

MAS 3105—Applied Linear Algebra, Spring 2018

(Clearly!) Print Name: ______________________________

Read all of what follows carefully before starting!

1. This test has **6 problems** and is worth **100 points**. *Please be sure you have all the questions before beginning!*

2. The exam is **closed-note** and **closed-book**. You may **not** consult with other students, and **no** calculators may be used!

3. Show all work clearly in order to receive full credit. **No work = no credit!** (unless otherwise stated)

4. You may use appropriate results from class and/or from the textbook as long as you fully and correctly state the result and where it came from.
   - If you use a result/theorem, you have to state *which* result you're using and explain *why* you're able to use it!

5. You **do not** need to simplify results, unless otherwise stated.

6. There is scratch paper at the end of the exam; you may also use the backs of pages or get more scratch paper from me.

7. Some questions are multiple choice.
   - Indicate correct answers by circling them and/or drawing a box around them.
   - More than one choice may be a correct answer for a question; if so, circle all correct answers!
   - There may be correct answers which aren't listed; in this case, only focus on the choices provided!

8. The notation I_n always denotes the $n \times n$ identity matrix. For example, $\mathsf{I}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Question	1 (10)	2 (10)	3 (30)	4 (20)	5 (20)	6 (10)	Total (100)
Points							

Do not write in these boxes! If you do, you get 0 points for those questions!

1. (*10 pts*) If $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, which of the following is equal to A^3?

(i). $\begin{pmatrix} 1 & 4 & 9 \\ 16 & 25 & 36 \end{pmatrix}$

(v). $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

(ii). $\begin{pmatrix} 1 & 8 & 27 \\ 64 & 125 & 196 \end{pmatrix}$

(vi). $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

(iii). $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(vii). $\begin{pmatrix} -1 & 1 & 2 \\ 4 & -2 & -1 \\ 3 & 11 & -2 \end{pmatrix}$

(iv). $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(viii). None of These

2. (*10 pts*) If $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, which of the following is equal to A^{T}?

(i). $\begin{pmatrix} 1 & 4 & 9 \\ 16 & 25 & 36 \end{pmatrix}$

(v). $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

(ii). $\begin{pmatrix} 1 & 8 & 27 \\ 64 & 125 & 196 \end{pmatrix}$

(vi). $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

(iii). $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(vii). $\begin{pmatrix} -1 & 1 & 2 \\ 4 & -2 & -1 \\ 3 & 11 & -2 \end{pmatrix}$

(iv). $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(viii). None of These

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^5$ be the transformation $T : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} 0 \\ x_1 - x_3 \\ x_2 \\ -x_1 \\ 0 \end{pmatrix}$.

(a) (*8 pts*) **True or False:** T is a linear transformation. Justify your claim.

SOLUTION:

Question 3(b) is on the next page

(b) (*2 pts*) What is the domain of T?

(c) (*2 pts*) What is the codomain of T?

(d) (*2 pts*) Is the codomain of T equal to the range of T? How do you know? If they *aren't* the same, find a point in codomain(T) that *isn't* in range(T).

Question 3(e) is on the next page

(e) (*8 pts*) Is T injective/one-to-one? Justify your claim.

(f) (*8 pts*) Is T surjective/onto? Justify your claim.

4. Let $S : \mathbb{R}^2 \to \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the transformations

$$S : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_3 - x_1 \end{pmatrix} \qquad \text{and} \qquad T = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \longmapsto \begin{pmatrix} 0 \\ y_2 \\ -y_1 \\ 3y_3 \end{pmatrix},$$

respectively.

(a) (*5 pts*) Find the canonical matrix A corresponding to the transformation S such that $S(\mathbf{x}) = \mathsf{A}\mathbf{x}$ for all $\mathbf{x}$ or state that no such matrix exists.

(b) (*5 pts*) Find the canonical matrix B corresponding to the transformation T such that $T(\mathbf{x}) = \mathsf{B}\mathbf{x}$ for all $\mathbf{x}$ or state that no such matrix exists.

Question 4(c) is on the next page

(c) (*10 pts*) Find the canonical matrix corresponding to the composition $T \circ S : \mathbb{R}^2 \to \mathbb{R}^4$ <u>or</u> state that no such matrix exists. **Hint**: Recall that $(T \circ S)(\mathbf{x}) = T(S(\mathbf{x}))$ for all $\mathbf{x}$.

SOLUTION:

5. Let $\mathsf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$.

(a) (*5 pts*) Find $\det(\mathsf{A})$.

SOLUTION:

Question 5(b) is on the next page

(b) (*10 pts*) Find A^{-1} or state that no such matrix exists. Justify your claim.

SOLUTION:

Question 5(c) is on the next page

(c) (*5 pts*) Use the result from part (b) to solve the linear system

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ 4x_1 & + & 5x_2 & + & 6x_3 & = & 0 \\ 7x_1 & + & 8x_2 & & & = & 2 \end{array}$$

<u>or</u> state that no solution exists.

SOLUTION:

6. (*1 pt ea.*) Indicate whether each of the following questions is True or False by writing the words "True" or "False". **No justification is required, and no credit will be given if you write *only* the letters "T" or "F"!**

   (a) An $n \times n$ matrix A may have more than one inverse. In other words, there may exist matrices B and C with $\mathsf{B} \neq \mathsf{C}$ such that $\mathsf{AB} = \mathsf{I}_n = \mathsf{BA}$ and $\mathsf{AC} = \mathsf{I}_n = \mathsf{CA}$.

   (b) If $T(\mathbf{x}) = \mathsf{A}\mathbf{x}$ is a linear transformation $\mathbb{R}^n \to \mathbb{R}^n$ and if $\det(\mathsf{A}) = 13$, then there is a linear transformation $S : \mathbb{R}^n \to \mathbb{R}^n$ such that $S(T(\mathbf{x})) = \mathbf{x} = T(S(\mathbf{x}))$ for all $\mathbf{x} \in \mathbb{R}^n$.

   (c) Every injective linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ has an inverse linear transformation.

   (d) If the $n \times n$ matrix A has linearly independent columns, then $\det(\mathsf{A})$ may equal zero.

   (e) If A and B are both nonsingular, then $(\mathsf{AB})^{-1} = \mathsf{A}^{-1}\mathsf{B}^{-1}$.

Question 6(f) is on the next page

(f) If $\det(\mathsf{A}) = 10$ and $\det(\mathsf{B}) = -1$, then $\det(\mathsf{AB}) = -10 = \det(\mathsf{BA})$.

(g) If the columns of an $n \times n$ matrix A span $\mathbb{R}^n$, then $\det(\mathsf{A})$ may equal zero.

(h) If A is invertible, then A^T is invertible.

(i) If A^T is invertible, then A is invertible.

(j) If $\det(\mathsf{A}) \neq 0$, then $\det\left(\mathsf{A}^{-1}\right) = \dfrac{1}{\det(\mathsf{A})}$.