

# Exam 1

MAS 3105—APPLIED LINEAR ALGEBRA, SPRING 2018

(CLEARLY!) PRINT NAME: \_\_\_\_\_

## Read all of what follows carefully before starting!

- This test has **5 problems** and is worth **100 points**. *Please be sure you have all the questions before beginning!*
- The exam is **closed-note** and **closed-book**. You may **not** consult with other students, and **no** calculators may be used!
- Show all work clearly in order to receive full credit. **No work = no credit!** (unless otherwise stated)
- You may use appropriate results from class and/or from the textbook as long as you fully and correctly state the result and where it came from.
  - If you use a result/theorem, you have to state *which* result you're using and explain *why* you're able to use it!
- You **do not** need to simplify results, unless otherwise stated.
- There is scratch paper at the end of the exam; you may also use the backs of pages or get more scratch paper from me.
- Some questions are True/False, and unless otherwise stated:
  - If you write *True*, you should give a “proof” or (thorough!) explanation of why.
 

**Example:** “All quadratic functions have derivatives which are linear” is *True*, and the proof is: If  $f(x) = ax^2 + bx + c$ , then  $f'(x) = 2ax + b$  is linear.
  - If you write *False*, you should give and explain a counterexample.
 

**Example:** “All polynomials have graphs which are parabolas” is *False*; a counterexample is the function  $f(x) = x^3$ , whose graph *isn't* a parabola, and I could “explain” why this is a counterexample by drawing the non-parabola graph of  $y = f(x)$ .
- The notation  $(\mathbf{v}_1 \mid \cdots \mid \mathbf{v}_n)$  always denotes the matrix whose columns are the vectors  $\mathbf{v}_1, \dots, \mathbf{v}_n$ .

Question	1 (35)	2 (20)	3 (10)	4 (20)	5 (15)	Total (100)
Points						

**Do not write in these boxes! If you do, you get 0 points for those questions!**

1. Let  $A = \begin{pmatrix} 1 & -1 & 2 & 4 \\ 0 & 3 & -1 & 1 \\ 1 & 1 & 0 & -2 \end{pmatrix}$  be an augmented matrix.

(a) (10 pts) Put  $A$  into Row Echelon Form (REF).

**Question 1(b) is on the next page**

(b) (5 pts) Write a system of linear equations associated to  $A$  using  $x_1, x_2$ , etc. as your variables.

(c) (5 pts) Is the system from part (b) consistent? Why or why not?

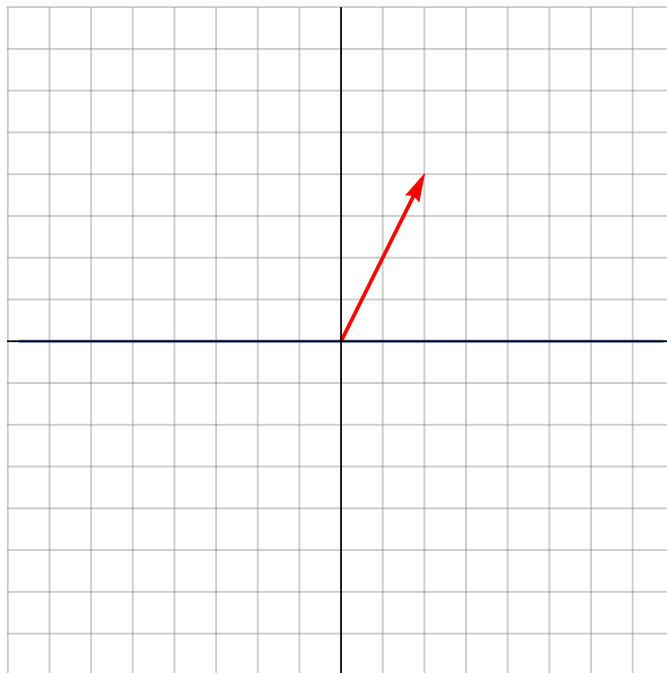
**Question 1(d) is on the next page**

(d) (5 pts) Find all solutions to the system from part (b) or state that no solutions exist.

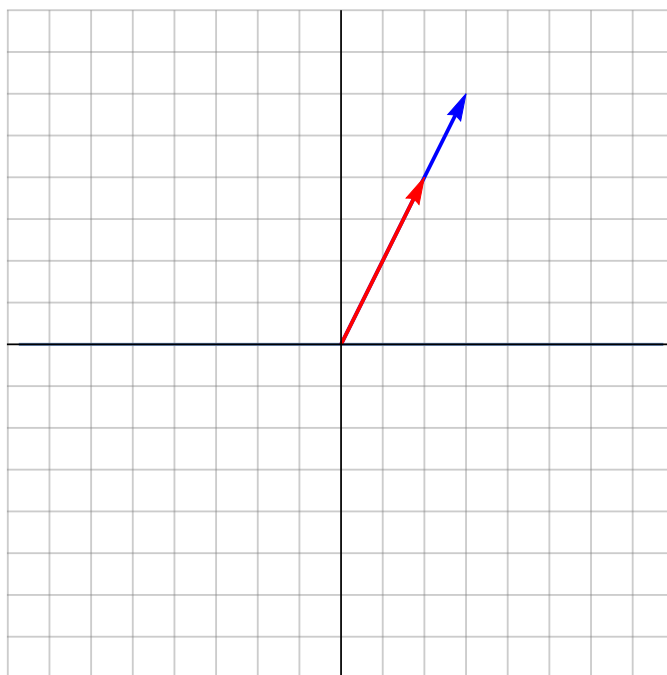
(e) (10 pts) **True or False:** A is row equivalent to  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ . Justify your answer!

2. (5 pts ea.) For each of the following, draw the indicated objects and write a brief description (less than once sentence) of what you've drawn. **The objects should be drawn on the same axes that are given!**

(a) Draw the span of the vector  $\mathbf{u}$  (shown in red).

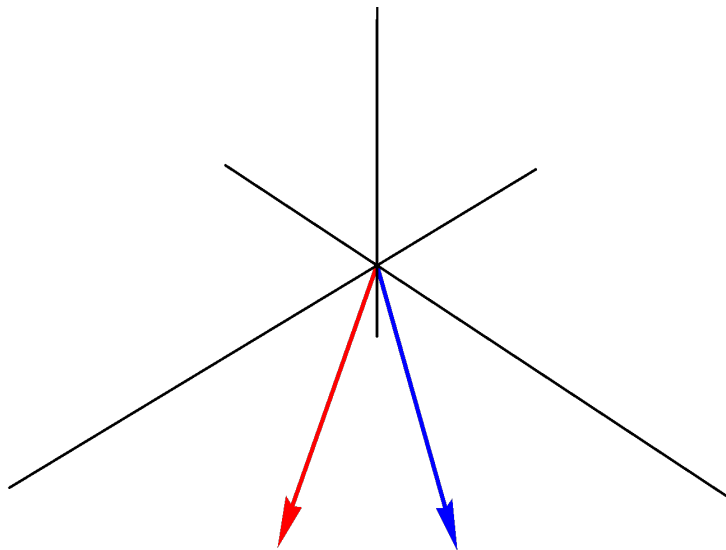


(b) Draw the span of the vectors  $\mathbf{u}$  (shown in red) and  $\mathbf{v}$  (shown in blue).

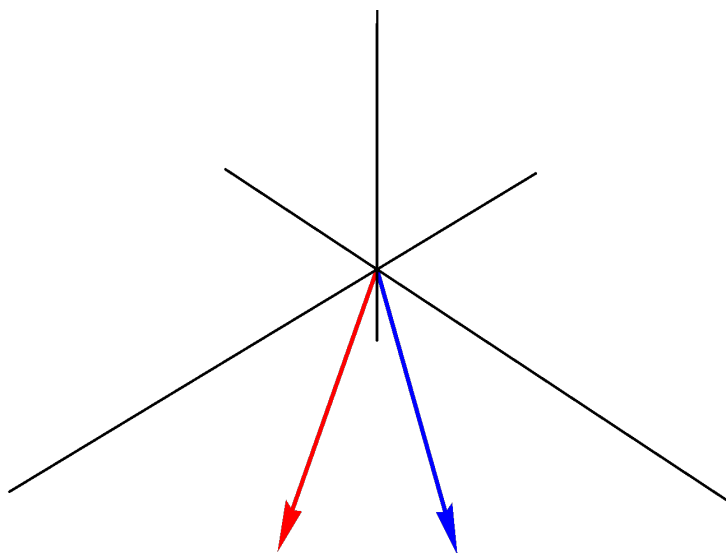


Question 2(c) is on the next page

(c) Draw the span of the vectors  $\mathbf{a}$  (shown in red) and  $\mathbf{b}$  (shown in blue).



(d) Draw a vector  $\mathbf{c}$  which makes the set  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$  linearly independent (where  $\mathbf{a}$  is shown in red and  $\mathbf{b}$  is shown in blue).



3. (2 pts ea.) Let  $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & 0 \end{pmatrix}$  and  $\mathbf{v} = \langle 1, -1, 1 \rangle$ . Provide an example matching each of the following criteria or state that no such example exists. Justify your answer!

(a) Give an example of a matrix  $\mathbf{C}$  such that the product  $\mathbf{BC}$  exists but the product  $\mathbf{CB}$  does not exist.

(b) Give an example of a matrix  $\mathbf{D}$  such that both products  $\mathbf{BD}$  and  $\mathbf{DB}$  exist but  $\mathbf{BD} \neq \mathbf{DB}$ .

(c) Give an example of a vector  $\mathbf{w}$  such that the dot product  $\mathbf{v} \cdot \mathbf{w}$  exists and equals 7.

(d) Give an example of a vector  $\mathbf{a}$  such that the vectors  $\{\mathbf{a}, \mathbf{v}\}$  are linearly independent.

(e) Give an example of a vector  $\mathbf{b}$  such that the vectors  $\{\mathbf{v}, \mathbf{b}\}$  are linearly dependent.

4. (10 pts ea.) Let  $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & h \end{pmatrix}$ .

(a) For which value(s)  $h$  does  $\mathbf{Ax} = \mathbf{0}$  have only the trivial solution?

(b) For which value(s)  $h$  does  $\mathbf{Ax} = \mathbf{0}$  have nontrivial solutions? In this case, express the solutions in terms of one or more free variables.



5. (3 pts ea.) Indicate whether each of the following questions is True or False by writing the words “True” or “False”. **No justification is required, and no credit will be given if you write *only* the letters “T” or “F”!**

(a) The columns of the matrix  $A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 0 & -2 \\ 5 & 0 & 3 \end{pmatrix}$  form a linearly independent set.

(b) Any linear combination of vectors can always be written as the product  $A\mathbf{v}$  for a suitable matrix  $A$  and a suitable vector  $\mathbf{v}$ .

(c) The solution set of a linear system whose augmented matrix is  $(\mathbf{a}_1 \mid \mathbf{a}_2 \mid \mathbf{a}_3 \mid \mathbf{b})$  is the same as the solution set of  $A\mathbf{x} = \mathbf{b}$  where  $A = (\mathbf{a}_1 \mid \mathbf{a}_2 \mid \mathbf{a}_3)$ .

(d) The equation  $A\mathbf{x} = \mathbf{0}$  may have zero solutions, one solution, or infinitely many solutions.

(e) If the sets  $\{\mathbf{u}, \mathbf{v}\}$  and  $\{\mathbf{v}, \mathbf{w}\}$  are each linearly independent, then the set  $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  is also linearly independent.

**Bonus:** A *Givens rotation* is a linear transformation from  $\mathbb{R}^n$  to  $\mathbb{R}^n$  used in computer programming to create zero entry in a vector. This transformation is given by multiplication by certain matrices.

(a) (3 pts) A Givens rotation in  $\mathbb{R}^2$  comes from multiplication by a matrix of the form  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ , where

$a^2 + b^2 = 1$ . Find  $a$  and  $b$  such that  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$  is rotated into  $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ .

(b) (7 pts) The following equation describes a Givens rotation in  $\mathbb{R}^3$ . Find  $a$  and  $b$ .

$$\begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2\sqrt{5} \\ 3 \\ 0 \end{pmatrix}, \text{ where } a^2 + b^2 = 1.$$

SOLUTION:

## Scratch Paper