Quiz 5/test prep 1 (front and back) Name: ____________________

Date Due: Monday, March 27

1. Rewrite each of the following regions in terms of polar coordinates.

   (i) $R_1 = \{(x, y) : 1 \leq x^2 + y^2 \leq 9 \text{ and } 0 \leq y \leq -x\}$

   (ii) $R_2 =$ the region in the first quadrant bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2y$.

(iii) R_3 = the region bounded between the circle $x^2 + y^2 = 1$, the curve defined implicitly as

$$5\sqrt{x^2+y^2} = 10 + \sin\left(10 \arctan\left(\frac{y}{x}\right)\right),$$

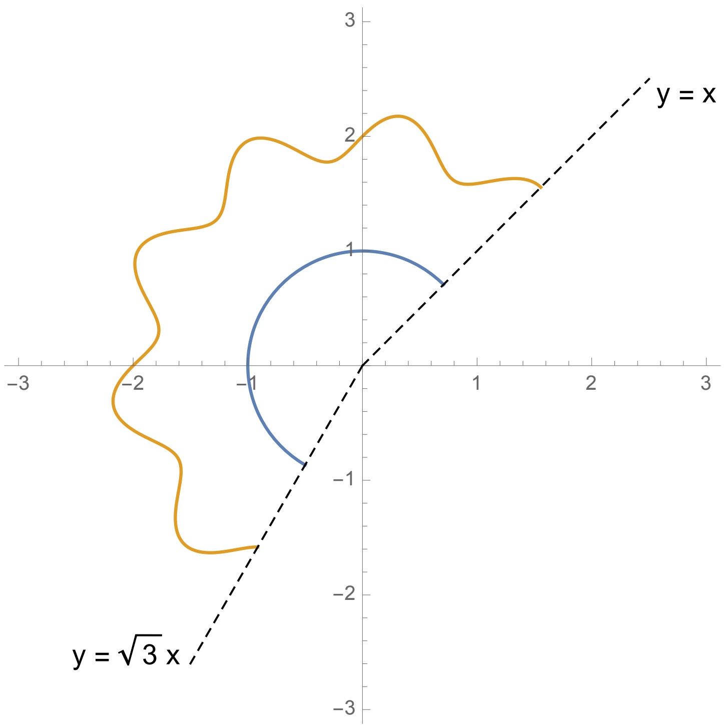
and the lines $y = x$ in quadrant 1 and $y = \sqrt{3}\,x$ in quadrant 3 (see below).

Hints:

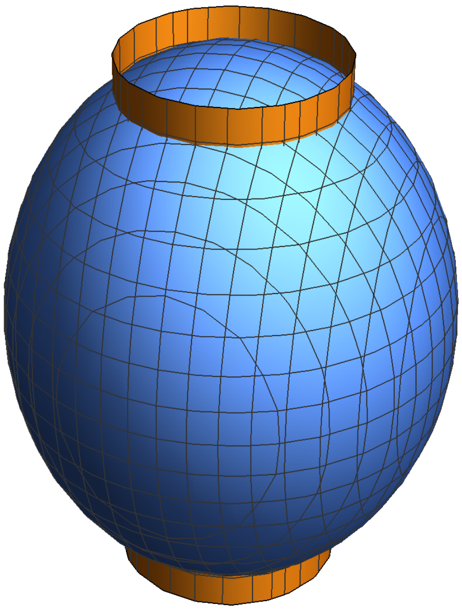
(a) Use $x = r\cos(\theta)$ and $y = r\sin(\theta)$ to solve for $\tan(\theta)$ and *then* use what you know about $\tan(\theta)$ to figure out what angles the two lines determine;

(b) figure out the $r = \ldots$ form of the (scary-looking) implicit curve by first doing direct substitution of x and y, and then solving for r (via lots of algebra); and

(c) Do your work on scratch paper!



2. Use polar coordinates to find the volume inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$ (see below).



SOLUTION:

3. Find the surface area of the part of the surface $z = 1 + 2x + 3y^2$ that lies above the triangle with vertices $(0, 0)$, $(0, 1)$, and $(3, 1)$.

SOLUTION:

4. Evaluate

$$\iiint\limits_E 3xy\,dV,$$

where E is the region under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

SOLUTION:

5. Use cylindrical coordinates to evaluate

$$\iiint_E x^2 \, dV,$$

where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 1$, and below the cone $z^2 = 3x^2 + 3y^2$.

SOLUTION: