Quiz 4 (front and back) Name: ~~KEY~~

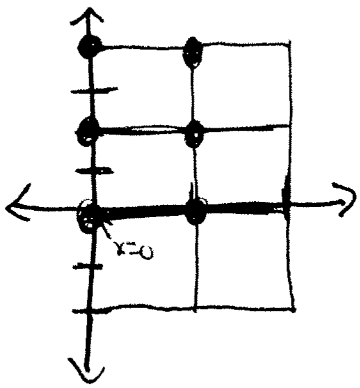
Throughout, let $f(x,y) = 1 - xy^2$.

1. If $R = [0,2] \times [-2,4]$, use a Riemann sum to estimate the value of

$$\iint_R f(x,y)\, dA$$

with $m = 2$, $n = 3$, and sample points equal to the top left corners of the rectangle.

SOLUTION:



Sample pts: $(0,0),\ (0,2),\ (0,4),\ (1,0),\ (1,2),\ (1,4)$

width: $\Delta x = \frac{2-0}{2} = 1,\quad \Delta y = \frac{4-(-2)}{3} = \frac{6}{3} = 2$

$\Rightarrow \Delta A = \Delta x \Delta y = 2$

So: $\iint_E \dots dA \approx \left[f(0,0) + f(0,2) + f(0,4) + f(1,0) + f(1,2) + f(1,4)\right]\Delta A$

$= [1 + 1 + 1 + 1 + (-3) + (-15)](2)$

$= (-14)(2) = -28.$

2. Find the exact value of

$$\iint_R f(x,y)\,dA$$

using iterated integrals/Fubini's theorem.

SOLUTION:

$$\iint_R \dots dA \underset{\text{fubini}}{=} \int_0^2 \int_{-2}^4 1 - xy^2 \, dy \, dx$$

$$= \int_0^2 \left(y - \tfrac{1}{3}xy^3 \Big]_{y=-2}^{y=4} \right) dx$$

$$= \int_0^2 \left(4 - \tfrac{64}{3}x - \left(-2 - \tfrac{-8}{3}x \right) \right) dx$$

$$= \int_0^2 6 - \tfrac{72}{3}x \quad dx$$

$$= \int_0^2 6 - 24x \, dx$$

$$= 6x - 12x^2 \Big]_{x=0}^{x=2}$$

$$= 12 - 48$$

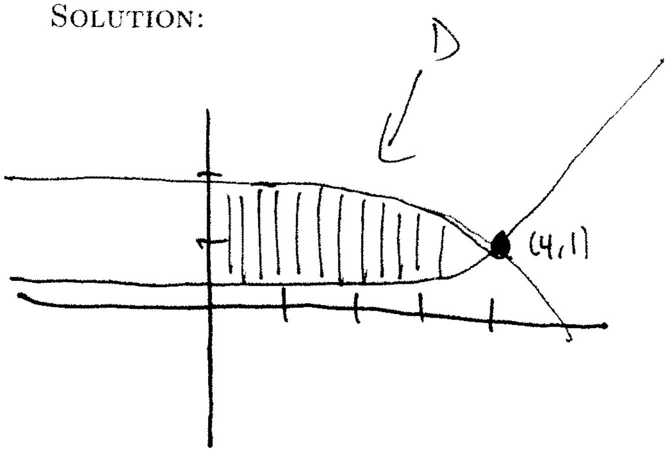
$$\boxed{= -36}$$

3. Find the exact value of

$$\iint_D f(x,y)\,dA$$

where D is the region in the first quadrant bounded by the y-axis and the curves $\boxed{y = e^{x-4}}$ and $\boxed{y = 2 - e^{x-4}.}$

Solution:



$y: e^{x-4} \to 2 - e^{x-4}$

$x: 0 \to 4$

$$\text{So: } \iint_D f(x,y)\,dA = \int_0^4 \int_{e^{x-4}}^{2-e^{x-4}} 1 - xy^2 \,dy\,dx = \int_0^4 \left(y - \frac{1}{3}xy^3 \Big]_{y=e^{x-4}}^{y=2-e^{x-4}} \right) dx$$

$$= \int_0^4 (2-e^{x-4}) - \frac{1}{3}x(2-e^{x-4})^3 - e^{x-4} + \frac{1}{3}x(e^{x-4})^3\,dx$$

$$= \int_0^4 2 - e^{x-4} - \frac{1}{3}x\left(8 + 3(4)(-e^{x-4}) + 3(2)(-e^{x-4})^2 - e^{3x-12}\right) - e^{x-4} + \frac{1}{3}xe^{3x-12}\,dx$$

$$= \int_0^4 2 - \frac{8}{3}x - 2e^{x-4} + \underbrace{4xe^{x-4}}_{\substack{\text{IBP}\\ u=x \quad v=e^{x-4}\\ u'=1 \quad v'=e^{x-4}}} - \underbrace{2xe^{2x-8}}_{\substack{\text{IBP}\\ u=x \quad v=\frac{1}{2}e^{2x-8}\\ u'=1 \quad v'=e^{2x-8}}} + \underbrace{\frac{2}{3}xe^{3x-12}}_{\substack{\text{IBP}\\ u=x \quad v=\frac{1}{3}e^{3x-12}\\ u'=1 \quad v'=e^{3x-12}}}\,dx$$

$$= 2x - \frac{8}{6}x^2 - 2e^{x-4} + 4e^{x-4}(x-1) - \frac{1}{2}e^{2x-8}(2x-1) + \frac{2}{27}e^{3x-12}(3x-1)\Big]_{x=0}^{x=4}$$

$$= \frac{-325}{54} + \frac{2}{27}e^{-12} - \frac{1}{2}e^{-8} + 6e^{-4}.$$