Quiz 3 (front and back) Name: ______________________

Note: A function f can have $\pm\infty$ as a limit, but if f approaches $+\infty$ along one curve/path and approaches $-\infty$ along another, the overall limit fails to exist.

1. Show that each of the following limits fails to exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2-y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{x^2-y^2}$

2. Let c be a constant and define $f(x,y)$ as follows:

$$f(x,y) = \begin{cases} \dfrac{\sin xy}{xy} & \text{if } (x,y) \neq (0,0) \\ c & \text{if } (x,y) = (0,0) \end{cases}$$

   Find the value c so that f is continuous on $\mathbb{R}^2$.

3. Use the limit definition of partial derivatives to find f_x and f_y for $f(x) = 3x^2 + xy - y^2$.

4. Define f, g, and h as follows:

$$f(x, y) = e^{x+y} \sin x \qquad g(x, y, z) = x \cos(y \cos z),$$

$$h(w, x, y, z) = e^w + e^x + e^y + e^z + ze^{w(xy+y^2)-x}.$$

Find each of the indicated partial derivatives. **Note**: If possible, it may be beneficial to change the order of the partials!

(a) f_{xy}.

(b) g_{yxzx}

(c) h_{wz}

(d) $h_{xywzxyxwz}$

5. Let $f(x,y) = 2x - 3x^2y + y^4$.

(a) Is f differentiable at the point $(1,4)$? Why or why not?

(b) Find the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(1,1,0)$.

(c) Find Δz and dz (as functions), where

$$\Delta z = f(a+\Delta x, b+\Delta y) - f(a,b) \qquad \text{and} \qquad dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy.$$

(d) Using part (c), compare the values of Δz and dz if x changes from 1 to 0.94 and y changes from 1 to 1.08.