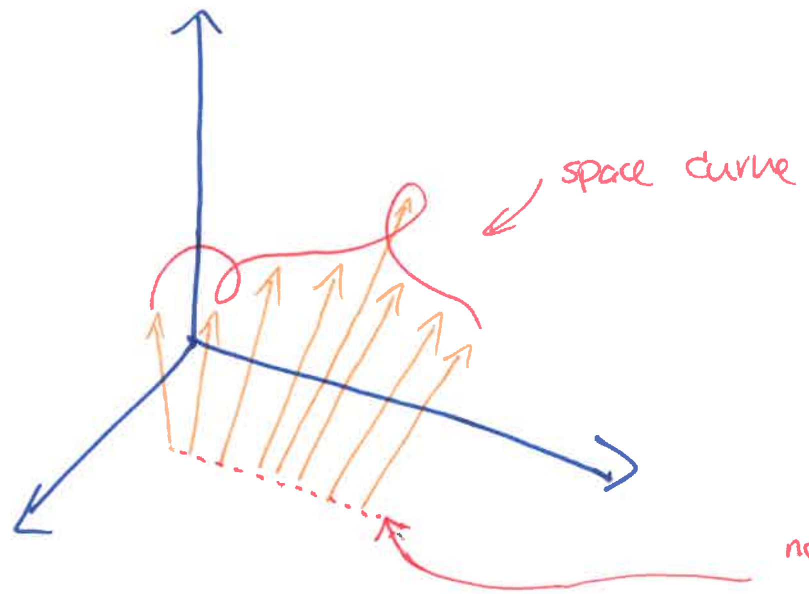
§16.6 - Parametric Surfaces

Recall: Space curves can be traced out by vector functions

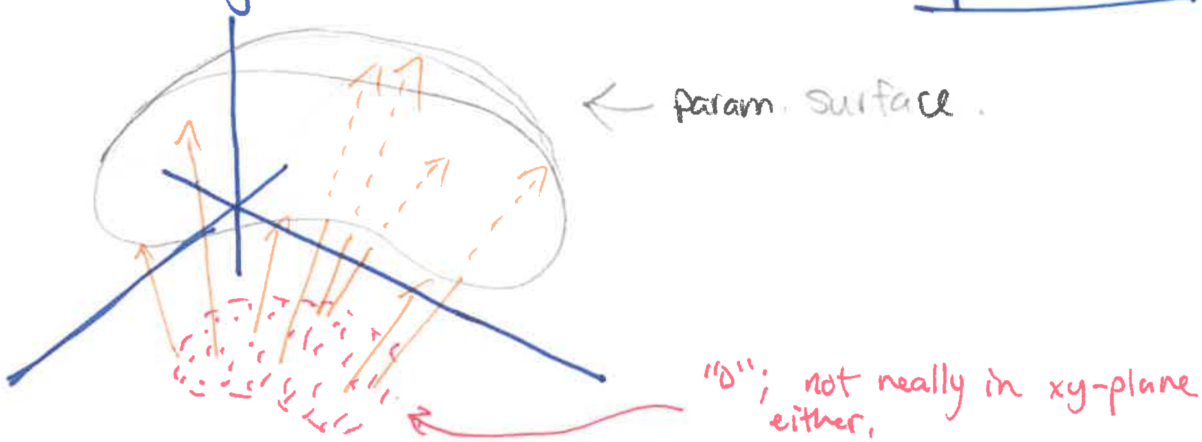
$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$.

↳ The idea is that a vector moves along a "t-axis" (1D) & its tip traces out a curve.



Now, if we let the vector func. $\vec{r}$ have <u>two</u> parameters, we can get a 2D shape (i.e. a surface!)

- If $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$ is defined on a region D in the uv-plane, the set $(x(u,v), y(u,v), z(u,v)) \in \mathbb{R}^3$ as (u,v) varies throughout D is called a <u>parametric surface</u>.

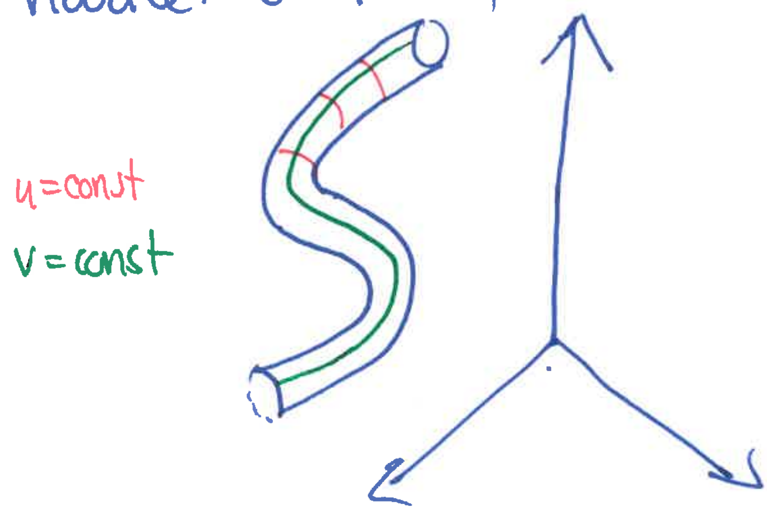


Ex: ① $\vec{r}(u,v) = \langle 2\cos u,\ v,\ 2\sin u \rangle$

↳ Cylinder (see Computer)

② $\vec{r}(u,v) = \langle (2+\sin v)\cos u,\ (2+\sin v)\sin u,\ u+\cos v \rangle$

↳ pasta noodle. (Computer)



③ Surfaces of revolution

↳ **Ex:** Rotate $y=f(x)$ about x-axis $(f(x)\geq 0,\ a\leq x\leq b)$.

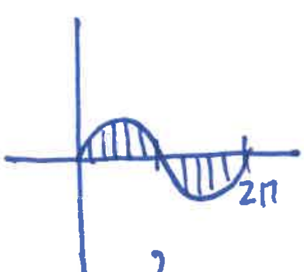
$\Rightarrow x=x \quad y=f(x)\cos\theta \quad z=f(x)\sin\theta$.

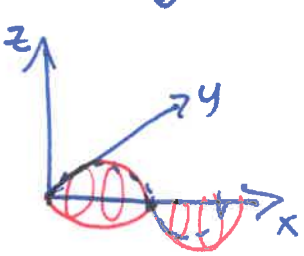
Ex: Rotate $y=\sin x$, $0\leq x\leq 2\pi$, about x-axis.

$\Rightarrow x=x \quad y=\sin x\cos\theta \quad z=\sin x\sin\theta$

↓

See computer





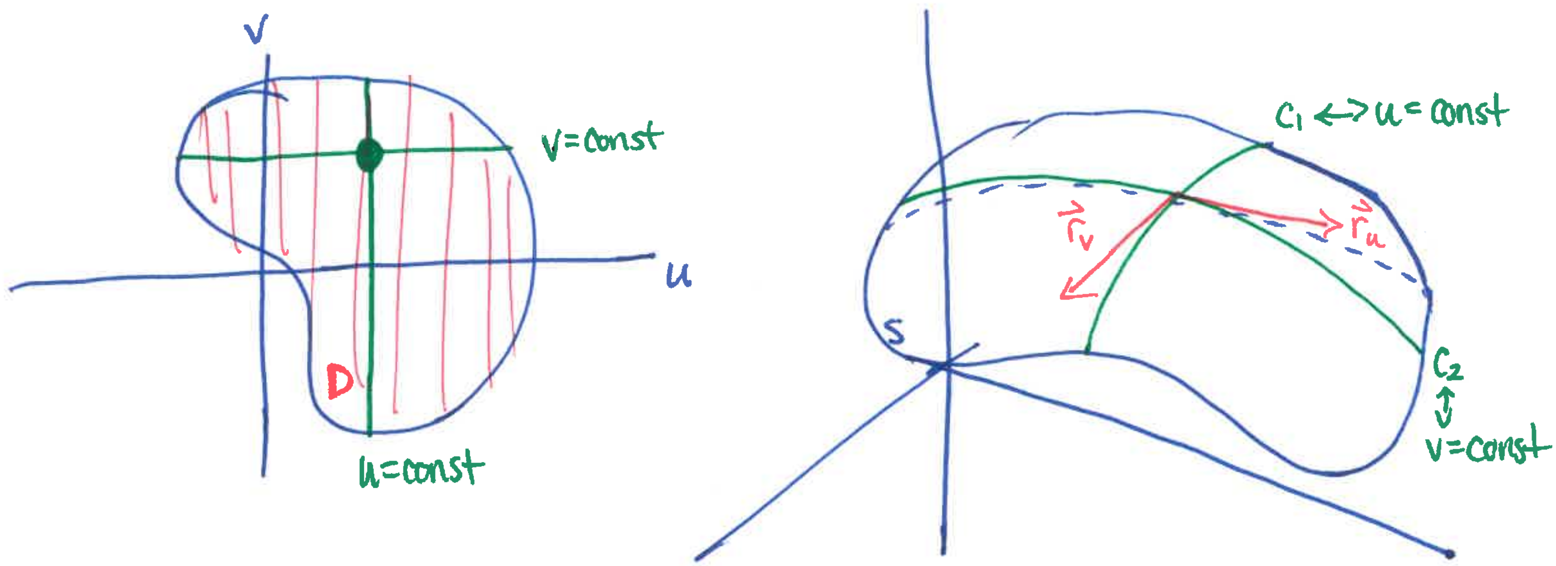
Note: Parametric Surfaces make better computer graphics!
(see sphere ex on cpu)

Tangent Planes

Given $\vec{r}(u,v) = \langle x(u,v),\ y(u,v),\ z(u,v) \rangle$, let

$$\vec{r}_u = \left\langle \frac{\partial x}{\partial u}(u,v),\ \frac{\partial y}{\partial u}(u,v),\ \frac{\partial z}{\partial u}(u,v) \right\rangle$$

$$\vec{r}_v = \left\langle \frac{\partial x}{\partial v}(u,v),\ \frac{\partial y}{\partial v}(u,v),\ \frac{\partial z}{\partial v}(u,v) \right\rangle.$$



- Tangent plane exists at a pt if $\vec{r}_u \times \vec{r}_v \neq \vec{0}$ at that pt.

  $\hookrightarrow$ If it exists, it is the plane containing $\vec{r}_u$ & $\vec{r}_v$ & normal to $\vec{r}_u \times \vec{r}_v$.

$u=1$ & $v=1$

<u>Ex:</u> Find tan. plane @ $(1,1,3)$ to surface $x=u^2,\ y=v^2,\ z=u+2v$.

- $\vec{r}_u = \langle 2u, 0, 1 \rangle$ & $\vec{r}_v = \langle 0, 2v, 2 \rangle \Rightarrow$ normal vec. is

$$\vec{r}_u \times \vec{r}_v = \langle -2v, -4u, 4uv \rangle.$$

@ pt $(u,v) = (1,1)$: $\langle -2, -4, 4 \rangle$.

- Plane is: $-2(x-1) - 4(x-1) + 4(x-3) = 0$.